Math 111, Section 01, Fall 2014

Worksheet 11, Tuesday, December 9, 2014

1. Evaluate each of the following limits. Please justify your answers. Be clear if the limit equals a value, $+\infty$, or $-\infty$, or Does Not Exist.

   (a) $\lim\limits_{x\to 5} \dfrac{5-x}{\sqrt{x+4}-3}$

   (b) $\lim\limits_{x\to 2} \dfrac{g(x^2)+x-3}{[g(x+1)]^2-x+2}$ where $g(x)=x-3$.

   (c) $\lim\limits_{x\to 1} \dfrac{x^2-8x+7}{x^2-2x+1}$

   (d) $\lim\limits_{x\to 5} \dfrac{x^2-4x-5}{|5-x|}$

2. Prove that $\lim\limits_{x\to 3} 5-2x=-1$ using the $\varepsilon-\delta$ definition of the limit.

3. Let $f(x)=\dfrac{3-x}{x+7}$. Compute the derivative in two different ways:

   (a) using the Limit Definition of the derivative

   (b) using the Quotient Rule.

4. Compute $\dfrac{dy}{dx}$ where $y=x^x$.

5. Compute each of the following integrals:

   (a) $\displaystyle\int \tan x \, dx.$

   (b) $\displaystyle\int_0^{\ln 2} \frac{e^{3x}}{\sqrt{8+e^{3x}}}\, dx.$

   (c) $\displaystyle\int_{e^3}^{e^9} \frac{1}{5x}\, dx.$

   (d) $\displaystyle\int_e^{e^4} \frac{3}{x\sqrt{\ln x}}\, dx.$

   (e) $\displaystyle\int \frac{1}{x(1+\ln x)}\, dx$

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Recall from class that the formula for Volumes of Revolution using the **Disk Method** and rotating about the x-axis was:

$$V = \int_a^b \pi \text{ (radius)}^2 \, dx$$

6. Let R be the region bounded by $y = e^x$, the x-axis, $x = 0$, and $x = 2$. Compute the volume of the solid formed by rotating R about the x-axis. Sketch the solid as well as one of the approximating disks.

   Note: You should sketch both the 2 and 3-dimensional sketches.

   Hint: To sketch one of the approximating disks, first sketch the approximating rectangle (from Area-Riemann sums days) in the 2-dimensional sketch. Then think about how that approximating rectangle spins around the axis.

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Recall from class that the formula for Volumes of Revolution using the **Washer Method** and rotating about the x-axis was:

$$V = \int_a^b \pi \left[(\text{outer radius})^2 - (\text{inner radius})^2\right] dx$$

7. Let R be the region bounded by $y = e^x + 1$, $y = x + 1$, $x = 0$, and $x = 1$. Compute the volume of the solid formed by rotating R about the x-axis. Sketch the solid as well as one of the approximating washers.

Turn in your own solutions.