Math 111 Fall 2014 Answer Key WS#9 11-18-14

1.(a) $$\int \frac{\sin x}{\cos^5 x}\,dx = -\int \frac{1}{u^5}\,du = -\int u^{-5}\,du = \frac{-u^{-4}}{-4} + C = \frac{1}{4u^4} + C$$

$$= \boxed{\frac{1}{4\cos^4 x} + C}$$

$$\boxed{\begin{aligned} u &= \cos x \\ du &= -\sin x\,dx \\ -du &= \sin x\,dx \end{aligned}}$$

(b) $$\int_{\pi/18}^{\pi/9} \sec^2(3x)\,dx = \frac{1}{3}\int_{\pi/6}^{\pi/3} \sec^2 u\,du = \frac{1}{3}\tan u \Big|_{\pi/6}^{\pi/3}$$

$$\boxed{\begin{aligned} u &= 3x \\ du &= 3dx \\ \tfrac{1}{3}du &= dx \end{aligned}} \quad \boxed{\begin{aligned} x = \pi/18 &\Rightarrow u = \pi/6 \\ x = \pi/9 &\Rightarrow u = \pi/3 \end{aligned}}$$

$$= \frac{1}{3}\left[\tan\frac{\pi}{3} - \tan\frac{\pi}{6}\right]$$

$\tan\frac{\pi}{3} \to \sqrt{3}$, $\tan\frac{\pi}{6} \to \frac{1}{\sqrt{3}}$

$$= \frac{1}{3}\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right] = \frac{1}{3}\left[\frac{3}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right] = \boxed{\frac{2}{3\sqrt{3}}}$$

(c) $$\int \frac{1}{x^2}\sqrt{1-1/x}\,dx = \int \sqrt{u}\,du = \int u^{1/2}\,du = \frac{2}{3}u^{3/2} + C$$

$$= \boxed{\frac{2}{3}\left(1 - 1/x\right)^{3/2} + C}$$

$$\boxed{\begin{aligned} u &= 1 - 1/x \\ du &= \frac{1}{x^2}\,dx \end{aligned}}$$

(d) $$\int_2^4 \frac{x}{(3x^2-8)^2}\,dx = \frac{1}{6}\int_4^{40} \frac{1}{u^2}\,du = \frac{1}{6}\int_4^{40} u^{-2}\,du = \frac{1}{6}\left(\frac{u^{-1}}{-1}\right)\Big|_4^{40} = \frac{-1}{6u}\Big|_4^{40}$$

$$\boxed{\begin{aligned} u &= 3x^2 - 8 \\ du &= 6x\,dx \\ \tfrac{1}{6}du &= x\,dx \end{aligned}} \quad \boxed{\begin{aligned} x = 2 &\Rightarrow u = 4 \\ x = 4 &\Rightarrow u = 40 \end{aligned}}$$

$$= -\frac{1}{6}\left[\frac{1}{40} - \frac{1}{4}\right] = -\frac{1}{6}\left[\frac{1}{40} - \frac{10}{40}\right]$$

$$= +\frac{9}{6 \cdot 40} = \boxed{+\frac{3}{80}}$$

1. (e) $\displaystyle\int \frac{1}{\sqrt{x}(1+\sqrt{x})^9}\,dx = 2\int \frac{1}{u^9}\,du = 2\int u^{-9}\,du = 2\left(\frac{u^{-8}}{-8}\right)+C$

$$= -\frac{1}{4u^8}+C = \boxed{-\frac{1}{4(1+\sqrt{x})^8}+C}$$

$$\boxed{\begin{aligned} u &= 1+\sqrt{x} \\ du &= \frac{1}{2\sqrt{x}}\,dx \\ 2du &= \frac{1}{\sqrt{x}}\,dx \end{aligned}}$$

(f) $\displaystyle\int_{2/3}^{14/3} \frac{1}{\sqrt{3x+2}}\,dx = \frac{1}{3}\int_4^{16} \frac{1}{\sqrt{u}}\,du = \frac{1}{3}\int_4^{16} u^{-1/2}\,du = \frac{1}{3}\cdot 2u^{1/2}\Big|_4^{16}$

$$\boxed{\begin{aligned} u &= 3x+2 \\ du &= 3dx \\ \tfrac{1}{3}du &= dx \end{aligned}} \quad \boxed{\begin{aligned} x=2/3 &\Rightarrow u=4 \\ x=14/3 &\Rightarrow u=16 \end{aligned}}$$

$$= \frac{2}{3}\sqrt{u}\Big|_4^{16} = \frac{2}{3}\left[\cancelto{4}{\sqrt{16}} - \cancelto{2}{\sqrt{4}}\right]$$

$$= \frac{2}{3}(2) = \boxed{\frac{4}{3}}$$

(g) $\displaystyle\int \frac{\tan\sqrt{x}\,\sec^2\sqrt{x}}{\sqrt{x}}\,dx = 2\int u\,du = u^2+C = \boxed{\tan^2\sqrt{x}+C}$

$$\boxed{\begin{aligned} u &= \tan\sqrt{x} \\ du &= \sec^2\sqrt{x}\left(\frac{1}{2\sqrt{x}}\right)dx \\ 2du &= \frac{\sec^2\sqrt{x}}{\sqrt{x}}\,dx \end{aligned}}$$

(h) $\displaystyle\int x(1+x)^{2/3}\,dx = \int (u-1)\,u^{2/3}\,du = \int u^{5/3}-u^{2/3}\,du = 3/8\,u^{8/3} - \frac{3}{5}u^{5/3}+C$

$$\boxed{\begin{aligned} u &= 1+x \Rightarrow x = u-1 \\ du &= dx \end{aligned}}$$

invert

$$= \boxed{\frac{3}{8}(1+x)^{8/3} - \frac{3}{5}(1+x)^{5/3}+C}$$

2. $f(x) = \int f'(x)\,dx = \int \frac{\sec x \tan x}{\sqrt{\sec x + 8}}\,dx = \int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du$

$$\boxed{\begin{array}{l} u = \sec x + 8 \\ du = \sec x \tan x\,dx \end{array}}$$

$= 2u^{1/2} + C = 2\sqrt{\sec x + 8} + C$

Use initial condition $f(0)=7$

$f(0) = 2\sqrt{\underbrace{\sec 0}_{1} + 8} + C = 2\sqrt{9} + C = 6 + C \overset{\text{set}}{=} 7 \Rightarrow C = 1$

Finally, specific solution is

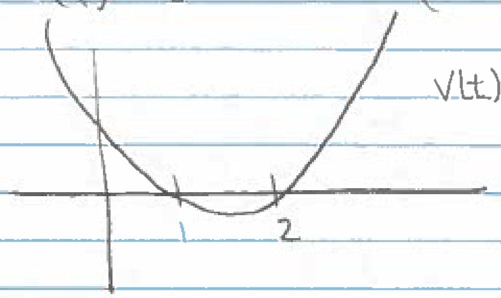
$$f(x) = \boxed{2\sqrt{\sec x + 8} + 1}$$

3. $g(x) = \int_x^9 \sqrt{1+\cos t}\,dt$

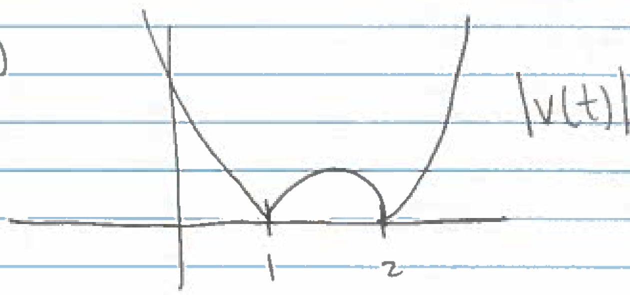
$g'(x) = \frac{d}{dx}\int_x^9 \sqrt{1+\cos t}\,dt = -\frac{d}{dx}\int_9^x \sqrt{1+\cos t}\,dt \overset{\text{FTC Part I}}{=} -\sqrt{1+\cos x}$

$g''(x) = \frac{d}{dx}\left[-\sqrt{1+\cos x}\right] = -\frac{1}{2\sqrt{1+\cos x}}(-\sin x) = \boxed{\frac{\sin x}{2\sqrt{1+\cos x}}}$

4. (a) $V(t) = t^2 - 3t + 2 = (t-2)(t-1)$



(b)



(c) $|v(t)| = \begin{cases} v(t) \text{ if } v(t) \geq 0 \\ \cancel{-v(t) \text{ if } v(t) < 0} \end{cases} = \begin{cases} t^2 - 3t + 2 & \text{if } t \leq 1 \text{ or } t \geq 2 \\ -t^2 + 3t - 2 & \text{if } 1 < t < 2 \end{cases}$

(d) $\text{Displacement} = \int_0^3 v(t)\,dt = \int_0^3 t^2 - 3t + 2\,dt = \frac{t^3}{3} - \frac{3t^2}{2} + 2t \Big|_0^3$

$= \left(9 - \frac{27}{2} + 6\right) - 0 = 15 - \frac{27}{2} = \frac{30}{2} - \frac{27}{2} = \boxed{\frac{3}{2}}$

(e) $\text{Total Distance} = \int_0^3 |v(t)|\,dt = \int_0^3 |t^2 - 3t + 2|\,dt$

28
99
127

$= \int_0^1 t^2 - 3t + 2\,dt + \int_1^2 -t^2 + 3t - 2\,dt + \int_2^3 t^2 - 3t + 2\,dt$

138
127
11

$= t^3/3 - 3t^2/2 + 2t \Big|_0^1 - t^3/3 + 3t^2/2 - 2t \Big|_1^2 + t^3/3 - 3t^2/2 + 2t \Big|_2^3$

$= \frac{1}{3} - 3/2 + 2 - (0 - 0 + 0) + \left(-8/3 + 6 - 4\right) - \left(-\frac{1}{3} + 3/2 - 2\right) + \left(9 - 27/2 + 6\right) - \left(\frac{8}{3} - 6 + 4\right)$

$= \frac{1}{3} - 3/2 + 2 - 8/3 + 2 + \frac{1}{3} - 3/2 + 2 + 15 - 27/2 - \frac{8}{3} + 2 = -14/3 - \frac{33}{2} + 23 = \boxed{\frac{11}{6}}$

$-\frac{28}{6} - \frac{99}{6} + \frac{138}{6}$