Math 111, Fall 2014

Answer Key

Worksheet 8, Tuesday, November 11th, 2014

2. Evaluate $\int_0^4 x - 1 \, dx$ using Riemann Sums and the Limit Definition of the Definite Integral.

Here $f(x) = x - 1$, $a = 0$, $b = 4$, $\Delta x = \frac{4-0}{n} = \frac{b-a}{n} = \frac{4}{n}$
and $x_i = a + x_i = 0 + i\left(\frac{4}{n}\right) = \frac{4i}{n}$.

$$\begin{aligned}
\int_0^4 x - 1 \, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \quad &= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{4i}{n}\right)\frac{4}{n} \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4i}{n} - 1\right)\frac{4}{n} \\
&= \lim_{n\to\infty} \left(\frac{4}{n}\sum_{i=1}^{n} \frac{4i}{n} - \frac{4}{n}\sum_{i=1}^{n} 1\right) \\
&= \lim_{n\to\infty} \left(\frac{16}{n^2}\sum_{i=1}^{n} i - \frac{4}{n}(n)\right) \\
&= \lim_{n\to\infty} \left(\frac{16}{n^2}\frac{n(n+1)}{2} - 4\right) \\
&= \lim_{n\to\infty} \left(8\left(\frac{n+1}{n}\right) - 4\right) \\
&= \lim_{n\to\infty} \left(8\left(\frac{n}{n} + \frac{1}{n}\right) - 4\right) \\
&= \lim_{n\to\infty} \left(8\left(1 + \frac{1}{n}\right) - 4\right) \\
&= 8 - 4 \\
&= \boxed{4}
\end{aligned}$$

3. Evaluate $\int_0^2 x^2 - 5x\ dx$ using Riemann Sums and the Limit Definition of the Definite Integral.

Here $f(x) = x^2 - 5x$, $a = 0$, $b = 2$, $\Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$ and $x_i = a + x_i = 0 + i\left(\frac{2}{n}\right) = \frac{2i}{n}$.

$$
\begin{aligned}
\int_0^2 x^2 - 5x\ dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x \quad &= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(\frac{2i}{n}\right)\frac{2}{n} \\
&= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\left(\frac{2i}{n}\right)^2 - 5\left(\frac{2i}{n}\right)\right)\frac{2}{n} \\
&= \lim_{n\to\infty}\left(\frac{2}{n}\sum_{i=1}^{n}\frac{4i^2}{n^2} - \frac{2}{n}\sum_{i=1}^{n}\frac{10i}{n}\right) \\
&= \lim_{n\to\infty}\left(\frac{8}{n^3}\sum_{i=1}^{n} i^2 - \frac{20}{n^2}\sum_{i=1}^{n} i\right) \\
&= \lim_{n\to\infty}\left(\frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{20}{n^2}\frac{n(n+1)}{2}\right) \\
&= \lim_{n\to\infty}\left(\frac{8}{6}\left(\frac{n}{n}\right)\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right) - \frac{20}{2}\left(\frac{n}{n}\right)\left(\frac{n+1}{n}\right)\right) \\
&= \lim_{n\to\infty}\left(\frac{4}{3}(1)\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) - 10(1)\left(1+\frac{1}{n}\right)\right) \\
&= \frac{8}{3} - 10 = \frac{8}{3} - \frac{30}{3} = \boxed{-\frac{22}{3}}
\end{aligned}
$$

4. Evaluate $\int_1^4 6 - 3x \; dx$ using Riemann Sums and the Limit Definition of the Definite Integral.

Here $f(x) = 6 - 3x,\; a = 1,\; b = 4,\; \Delta x = \dfrac{b-a}{n} = \dfrac{4-1}{n} = \dfrac{3}{n}$

and $x_i = a + i\Delta x = 1 + i\left(\dfrac{3}{n}\right) = 1 + \dfrac{3i}{n}$.

$$
\begin{aligned}
\int_1^4 6 - 3x \; dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \quad &= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(1 + \frac{3i}{n}\right)\frac{3}{n} \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left(6 - 3\left(1 + \frac{3i}{n}\right)\right)\frac{3}{n} \\
&= \lim_{n\to\infty} \left(\frac{3}{n}\sum_{i=1}^{n}\left(3 - \frac{9i}{n}\right)\right) \\
&= \lim_{n\to\infty} \left(\frac{3}{n}\left(\sum_{i=1}^{n} 3 - \sum_{i=1}^{n}\frac{9i}{n}\right)\right) \\
&= \lim_{n\to\infty} \left(\frac{9}{n}\sum_{i=1}^{n} 1 - \frac{27}{n^2}\sum_{i=1}^{n} i\right) \\
&= \lim_{n\to\infty} \left(\frac{9}{n}(n) - \frac{27}{n^2}\frac{n(n+1)}{2}\right) \text{ using } (*) \\
&= \lim_{n\to\infty} \left(9 - \frac{27}{2}\left(\frac{n}{n}\right)\left(\frac{n+1}{n}\right)\right) \\
&= \lim_{n\to\infty} \left(9 - \frac{27}{2}(1)\left(1 + \frac{1}{n}\right)\right) \\
&= 9 - \frac{27}{2} \\
&= \boxed{-\frac{9}{2}}
\end{aligned}
$$