Math 111, Section 01, Fall 2014

Answer Key Worksheet 7, Tuesday, October 28, 2014

Reminder: This worksheet is a chance for you not to just *do* the problems, but rather *understand* the problems. Please discuss ideas with your partners. Your solutions should be focused moreso on presentation than on numerical values.

Position–Velocity Problems

1. Suppose a falling ball's position is given by $s(t) = 256 - 16t^2$ feet at t seconds.

   (a) What is the balls initial position above the ground?
   Initial position is $\boxed{s(0) = 256}$ feet.

   (b) Find the average velocity of the ball during the initial three seconds of its drop.
   Average velocity$= V_{\text{ave}} = \dfrac{\text{change in distance}}{\text{change in time}} = \dfrac{s(3) - s(0)}{3 - 0} = \dfrac{(256 - 16(3)^2) - 256}{3} = \dfrac{-144}{3} = \boxed{-48}\dfrac{\text{ft}}{\text{sec}}$

   (c) Find the velocity at 2 seconds and 3 seconds respectively.
   $v(t) = s'(t) = -32t$ so $\boxed{v(2) = -64}\frac{\text{ft}}{\text{sec}}$ and $\boxed{v(3) = -96}\frac{\text{ft}}{\text{sec}}$
   Understand why the velocity is negative here. The ball is falling in the negatively oriented direction. So the position $s(t)$ is decreasing $\searrow$ which means its derivative $s'(t) = v(t)$ is negative.

   (d) How much time passed before the ball hit the ground?
   To find when the ball hits the ground, set $s(t) = 0$ and solve for time t.
   $s(t) = 256 - 16t^2 = 0 \implies t^2 = \frac{256}{16} \implies t = \pm 4 \implies \boxed{t = 4}$ seconds, since we are considering positive time here. So the ball hits the ground in 4 seconds.

   (e) What was the balls velocity when it hit the ground?
   The ball's velocity, when it hits the ground, is $v(4) = -32(4) = \boxed{-128}\frac{\text{ft}}{\text{sec}}$

   (f) Finally, find the balls acceleration at 3 seconds
   $a(t) = v'(t) = s''(t) = -32$ (note, it's constant here, acceleration due to gravity)
   So, $\boxed{a(3) = -32}\frac{\text{ft}}{\text{sec}^2}$

2. A man stands on the edge of a bridge over a river. He throws a stone straight upward in the air with an initial velocity of 64 feet per second. The ball reaches a height of $s(t) = -16t^2+64t+80$ feet in t seconds above the water. Answer the following questions:

   (a) What is the intitial height of the stone?
   Initial position is $s(0) = 80$ feet.

   (b) What is the maximum height the stone reaches?
   Max height occurs when $v(t) = 0$.
   Compute $v(t) = -32t + 64 \stackrel{\text{set}}{=} 0$ or when $t = 2$ seconds.
   Max height is $s(2) = -64 + 128 + 80 = 144$ feet above the water.

   (c) What is the stone's velocity at time $t = 1$ second?
   The stone's velicity at 1 second is given by $v(1) = -32 + 64 = 32$ ft/sec.

   (d) When is the stone 128 feet above the water?
   Set $s(t) = -16t^2 + 64t + 80 = 128$ and solve for t.
   Here $-16t^2 + 64t - 48 = -16(t^2 - 4t + 3) = -16(t-3)(t-1) = 0$ or when $t = 3$ or $t = 1$ seconds.
   The stone is 128 feet above the water at time 1 second and time 3 seconds.

   (e) What is the stone's acceleration at any time t?
   The stone's acceleration at any time t is given by $a(t) = -32$ ft/sec^2.

   (f) At what time will the stone hit the water?
   The stone hits the water when $s(t) = -16t^2 + 64t + 80 = 0$ which factors
   $-16(t^2 - 4t - 5) = -16(t-5)(t+1) = 0$ or when $t = 5$ or $t = -1$ (ignore negative time here).
   So the stone hits the water when 5 seconds has passed.

   (g) What is the stone's velocity when it hits the water?
   If the stone hits the water at 5 seconds, then the velocity at impact is
   $v(5) = -32(5) + 64 = -160 + 64 = -96$ feet per second.