Worksheet #6 Math 111 Fall 2014

1. (a) $\lim_{x\to\infty} \dfrac{8x^2-17}{3x^4+2014x+6}\dfrac{\left(\frac{1}{x^4}\right)}{\left(\frac{1}{x^4}\right)} = \lim_{x\to\infty} \dfrac{\frac{8x^2}{x^4}-\frac{17}{x^4}}{\frac{3x^4}{x^4}+\frac{2014x}{x^4}+\frac{6}{x^4}}$ must simplify

$= \lim_{x\to\infty} \dfrac{\overset{\to 0}{\cancel{8/x^2}} - \overset{\to 0}{\cancel{17/x^4}}}{3+\underset{\to 0}{\cancel{2014/x^3}}+\underset{\to 0}{\cancel{6/x^4}}} = \dfrac{0}{3} = \boxed{0}$

Note: could also use $1/x^2$

$\lim_{x\to\infty} \dfrac{8x^2-17}{3x^4+2014x+6}\dfrac{\left(1/x^2\right)}{\left(1/x^2\right)} = \lim_{x\to\infty} \dfrac{8-\overset{\to 0}{\cancel{17/x^2}}}{\underset{\to\infty}{x^2}+\underset{\to 0}{2014/x}+\underset{\to 0}{6/x^2}} = \boxed{0}$

(b) $\lim_{x\to\infty} \dfrac{x^7-4x+7}{x^2+9}\dfrac{\left(1/x^2\right)}{\left(\frac{1}{x^2}\right)} = \lim_{x\to\infty} \dfrac{\frac{x^7}{x^2}-\frac{4x}{x^2}+\frac{7}{x^2}}{\cancel{\frac{x^2}{x^2}}+9/x^2} = \lim_{x\to\infty} \dfrac{\underset{\to\infty}{x^5}-\overset{\to 0}{\cancel{4/x}}+\overset{\to 0}{\cancel{7/x^2}}}{1+\underset{\to 0}{\cancel{9/x^2}}} = \boxed{\infty}$

OR

$\lim_{x\to\infty} \dfrac{x^7-4x+7}{x^2+9}\dfrac{\left(\frac{1}{x^7}\right)}{\left(\frac{1}{x^7}\right)} = \lim_{x\to\infty} \dfrac{1-\overset{\to 0}{\cancel{4/x^6}}+\overset{\to 0}{\cancel{7/x^7}}}{\underset{\to 0}{\frac{1}{x^5}}+\underset{\to 0}{\cancel{9/x^7}}} = \dfrac{1}{0^+} = \boxed{\infty}$

2. (a) $f(x) = (9-x^2)^8(x^3-6x)^9$

$$f'(x) = \boxed{(9-x^2)^8\cdot 9(x^3-6x)^8[3x^2-6] + (x^3-6x)^9\cdot 8(9-x^2)^7(-2x)}$$

Do Not Simplify Here.

(b) $f(t) = \sin^3\left(\sec\left(\dfrac{1}{t^{7/8}}\right)\right) = \left[\sin\left(\sec\left(t^{-7/8}\right)\right)\right]^3$

$$f'(t) = \boxed{3\sin^2\left(\sec(t^{-7/8})\right)\cdot\cos\left(\sec(t^{-7/8})\right)\cdot\sec(t^{-7/8})\tan(t^{-7/8})\cdot\left(-\tfrac{7}{8}t^{-15/8}\right)}$$

2(c) $f(x) = \sqrt{\dfrac{\sin x}{x - \cos^2 x}}$

$$\boxed{f'(x) = \frac{1}{2\sqrt{\dfrac{\sin x}{x-\cos^2 x}}} \left[\frac{(x-\cos^2 x)(\cos x) - \sin x\,(1 - 2\cos x(-\sin x))}{(x-\cos^2 x)^2} \right]}$$

(2d) $f(x) = \dfrac{1}{(\tan(7x) + 1/x)^{5/7}} = \left(\tan(7x) + 1/x\right)^{-5/7}$

$$\boxed{f'(x) = -\frac{5}{7}\left[\tan(7x) + \frac{1}{x}\right]^{-12/7} \cdot \left(\sec^2(7x)\cdot 7 - 1/x^2\right)}$$

3. $G(x) = \dfrac{5x}{x^2+1}$ Closed Interval Method $[0,2]$

Step1: Find Critical Numbers $G'(x) = \dfrac{\overbrace{(x^2+1)5}^{5x^2+5} - \overbrace{(5x)(2x)}^{-10x^2}}{(x^2+1)^2} = \dfrac{5-5x^2}{(x^2+1)^2} \overset{\text{set}}{=} 0$

$\Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

Step2: Evaluate G at Critical Number(s)

$G(1) = \dfrac{5}{1+1} = \boxed{5/2}$ ⟵ Absolute Max Value

$\cancel{G(-1) = \dfrac{-5}{1+1} = -5/2}$ outside closed interval $[0,2]$

Step 3: Evaluate G at Endpoints.

$G(0) = \boxed{0}$ ⟵ Absolute Min Value

$G(2) = \dfrac{10}{4+1} = \dfrac{10}{5} = 2$

Step 4: Pick off Values (above)

4. $f(x) = \dfrac{x^2-9}{x^2-4}$

• Domain: $\{x \mid x \neq \pm 2\}$

• V.A. $x = \pm 2$

• H.A. $\displaystyle\lim_{x\to\pm\infty} \frac{x^2-9}{x^2-4} \cdot \frac{(1/x^2)}{(1/x^2)} = \lim_{x\to\pm\infty} \frac{1-\overset{\to 0}{\cancel{9/x^2}}}{1-\underset{\to 0}{\cancel{4/x^2}}} = 1 \Rightarrow$ H.A. @ $y=1$

• First Derivative. $f'(x) = \dfrac{10x}{\underbrace{(x^2-4)^2}_{\oplus}} \overset{\text{set}}{=} 0 \Rightarrow x = 0.$

• Sign Testing into f'

	-10	-2	-1	0	1	2	10
f'	$\ominus$		$\ominus$		$\oplus$		$\oplus$
f	$\searrow$	V.A.	$\searrow$	local min	$\nearrow$	V.A.	$\nearrow$

f Increasing $(0,2) \cup (2,\infty)$

Decreasing $(-\infty,-2) \cup (-2,0)$

Local Min $(0, f(0)) = (0, 9/4)$

Local Max None.

• Second Derivative. $f''(x) = \dfrac{-10(3x^2+4)}{(x^2-4)^3} \overset{\text{set}}{=} 0$ No Possible Inflection Pts.

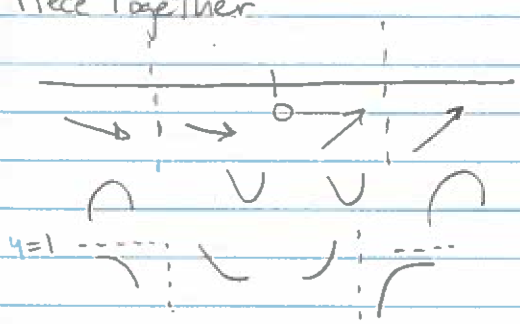
• Sign Testing into f''

	-10	-2	0	2	10
f''	$\ominus$		$\oplus$		$\ominus$
f	$\cap$	V.A.	$\cup$	V.A.	$\cap$

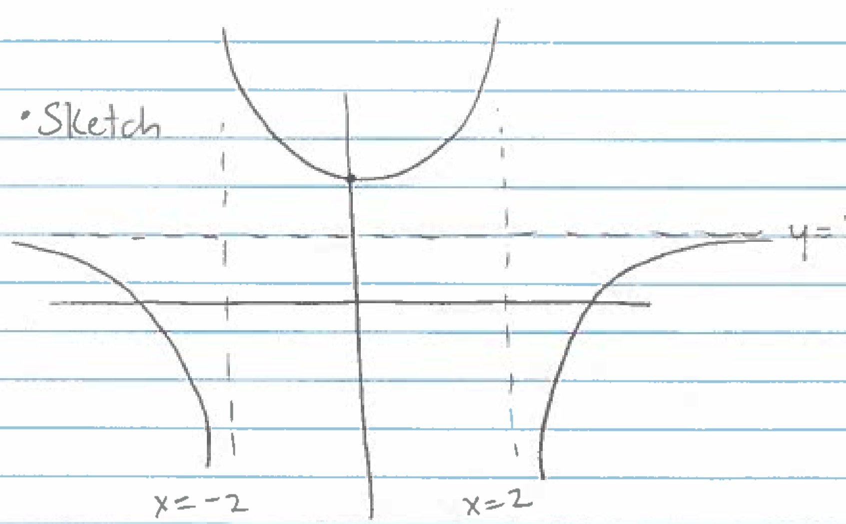
CU $(-2,2)$

CD $(-\infty,-2) \cup (2,\infty)$

• Piece Together



• Sketch



5. $y^3+\cos(xy) = 2+xy^2$

(a) $\frac{d}{dx}\left[y^3+\cos(xy)\right] = \frac{d}{dx}\left[2+xy^2\right]$

$$3y^2\frac{dy}{dx} - \sin(xy)\left[x\frac{dy}{dx}+y\right] = x\,2y\frac{dy}{dx} + y^2(1)$$

$$\underbrace{3y^2\frac{dy}{dx}} - \underbrace{x\sin(xy)\frac{dy}{dx}} - y\sin(xy) = \underbrace{2xy\frac{dy}{dx}} + y^2$$

$$\left[3y^2 - x\sin(xy) - 2xy\right]\frac{dy}{dx} = y^2 + y\sin(xy)$$

Solve $\frac{dy}{dx} = \boxed{\dfrac{y^2+y\sin(xy)}{3y^2-x\sin(xy)-2xy}}$

(b) $\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{1+1\cdot\cancel{\sin(0)}^{0}}{3-0-0} = \frac{1}{3}$

slope $=1/3$ Point $=(0,1)$

$y-1 = 1/3\,(x-0) \Rightarrow \boxed{y = 1/3\,x+1}$

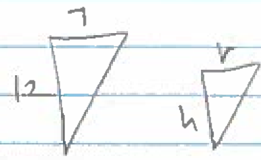
6(a) · Diagram

· Variables Let r = radius of water level at time t.
h = height of water level at time t.
V = Volume of water level at time t

Given $\frac{dr}{dt} = -2$ ft/min

$\frac{dV}{dt} = ?$ when $r = 2$ft.

· Equation $V = \frac{1}{3}\pi r^2 h$



Similar Triangles
$\frac{r}{7} = \frac{h}{12} \Rightarrow h = \frac{12}{7} r$

$V = \frac{1}{3}\pi r^2 \left(\frac{12}{7} r\right)$

$= \frac{4}{7}\pi r^3$

· Differentiate. $\frac{d}{dt}\left[V\right] = \frac{d}{dt}\left[\frac{4}{7}\pi r^3\right]$

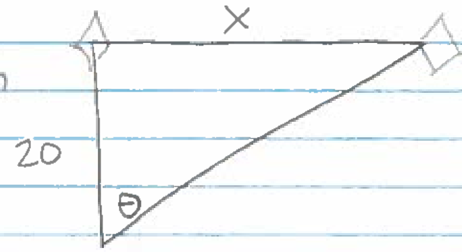
$\frac{dV}{dt} = \frac{12\pi}{7} r^2 \frac{dr}{dt}$

· Substitute
· Solve $\frac{dV}{dt} = \frac{12\pi}{7}(2)^2 \cdot (-2) = \boxed{\frac{-96\pi}{7}}$ ft³/min

Makes Sense: Volume Decreasing

· Answer The volume of the water is <u>decreasing</u> $\frac{96\pi}{7}$ cubic feet every minute.

(6(b)) · Diagram



· Variable Let x = distance the kite travelled horizontally

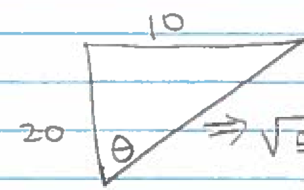
θ = angle formed by string and vertical.

Given $\frac{dx}{dt} = 5$ ft/sec. $\Rightarrow$ after 2 seconds, travelled 10 ft.

$\frac{d\theta}{dt} = ?$ when $x = 10$ ft.

· Equation $\tan\theta = \frac{x}{20}$

· Differentiate $\frac{d}{dt}\left[\tan\theta\right] = \frac{d}{dt}\left[\frac{x}{20}\right]$



$\Rightarrow \sqrt{500} = 10\sqrt{5}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{20}\frac{dx}{dt}$

$\sec\theta = \frac{10\sqrt{5}}{20} = \frac{\sqrt{5}}{2}$

· Substitute $\left(\frac{\sqrt{5}}{2}\right)^2 \frac{d\theta}{dt} = \frac{1}{20} \cdot 5$

· Solve $\frac{5}{\cancel{4}} \frac{d\theta}{dt} = \frac{1}{\cancel{4}}$

$\frac{d\theta}{dt} = \frac{1}{5}$ rad./sec.

· Answer: The angle between string and vertical is increasing at a rate of $\frac{1}{5}$ radians per second.