Math 111, Section 01, Fall 2014

Worksheet 6, Tuesday, October 21, 2014

1. Compute each of the following limits. Justify your answers.

(a) $\displaystyle\lim_{x\to\infty} \frac{8x^2 - 17}{3x^4 + 2014x + 6}$

(b) $\displaystyle\lim_{x\to\infty} \frac{x^7 - 4x + 7}{x^2 + 9}$

2. Differentiate each of the following functions. You **do not** need to simplify your answers. Please do not waste time simplifying your derivative.

(a) $f(x) = (9 - x^2)^8 (x^3 - 6x)^9$

(b) $f(t) = \sin^3\left(\sec\left(\frac{1}{t^{\frac{7}{8}}}\right)\right)$

(c) $f(x) = \sqrt{\dfrac{\sin x}{x - \cos^2 x}}$

(d) $f(x) = \dfrac{1}{\left(\tan(7x) + \frac{1}{x}\right)^{\frac{5}{7}}}$

3. Find the absolute maximum and absolute minimum value(s) of the function

$$G(x) = \frac{5x}{x^2 + 1} \quad \text{on the interval} \quad [0, 2].$$

4. Let $\boldsymbol{f(x) = \dfrac{x^2 - 9}{x^2 - 4}}$. For this function, discuss domain, vertical and horizontal asymptote(s), interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve.

Take my word for it that (you do **not** have to compute these)

$$\boldsymbol{f'(x) = \frac{10x}{(x^2 - 4)^2}} \quad \text{and} \quad \boldsymbol{f''(x) = \frac{-10(3x^2 + 4)}{(x^2 - 4)^3}}.$$

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5. Consider the curve given by $y^3 + \cos(xy) = 2 + xy^2$.

(a) First compute $\dfrac{dy}{dx}$.

(b) Next, find the equation of the tangent line to this curve at the point $(0, 1)$.

6. Related Rates

(a) A conical tank, 14 feet across the entire top and 12 feet deep, is leaking water. The radius of the water level is decreasing at the rate of 2 feet per minute. How fast is the water leaking out of the tank when the radius of the water level is 2 feet?

**Recall the volume of the cone is given by $V = \frac{1}{3}\pi r^2 h$

(b) A kite starts flying 20 feet directly above the ground. The kite is being blown horizontally at 5 feet per second. When the kite has blown horizontally for 2 seconds, how fast is the angle between the string and the vertical changing?

$\boxed{\textbf{Turn in your own solutions.}}$