WS#4 Math 111 Fall 2014

1. $f(x) = \sqrt{x}$

First, $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$\frac{0}{0}$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \left[\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right]$$

$$= \lim_{h \to 0} \frac{\cancel{x}+h - \cancel{x}}{h\left[\sqrt{x+h} + \sqrt{x}\right]}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}\left[\sqrt{x+h} + \sqrt{x}\right]}$$

$$= \frac{1}{\sqrt{x} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

Match!

Second, $f(x) = x^{1/2}$

$$f'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2x^{1/2}} = \boxed{\frac{1}{2\sqrt{x}}}$$

$$x^{25/30} x^{36/30} = x^{61/30}$$

2. $$f(x) = \frac{5}{6}x + x^{5/6} + \frac{1}{x^{5/6}} + \frac{5}{6x} + 5x^6 - \frac{1}{6x^5} + x^{5/6} \cdot x^{6/5}$$

$$= \frac{5}{6}x + x^{5/6} + x^{-5/6} + \frac{5}{6}x^{-1} + 5x^6 - 6x^{-5} + x^{61/30}$$

$$f'(x) = \boxed{\frac{5}{6} + \frac{5}{6}x^{-1/6} - \frac{5}{6}x^{-11/6} - \frac{5}{6}x^{-2} + 30x^5 + 30x^{-6} + \frac{61}{30}x^{31/30}}$$

3. $f(x) = 5x^2 - \sqrt{x} + x^{5/7} + \frac{3}{x} - \frac{2}{x^2} + \frac{1}{\sqrt{x}}$

$= 5x^2 - x^{1/2} + x^{5/7} + 3x^{-1} - 2x^{-2} + x^{-1/2}$

$f'(x) = \boxed{10x - \frac{1}{2\sqrt{x}} + \frac{5}{7}x^{-2/7} - 3x^{-2} + 4x^{-3} - \frac{1}{2}x^{-3/2}}$

4. $f(x) = (x^4 - 5x^3 + 6)\sqrt{x}$ Use Algebra to set up Power Rules

$= (x^4 - 5x^3 + 6)(x^{1/2})$

$= x^{9/2} - 5x^{7/2} + 6x^{1/2}$

$f'(x) = \boxed{\frac{9}{2}x^{7/2} - \frac{35}{2}x^{5/2} + 3x^{-1/2}}$

OR $f(x) = (x^4 - 5x^3 + 6)\sqrt{x}$ Use Product Rule.

$f'(x) = (x^4 - 5x^3 + 6)\left[\frac{1}{2\sqrt{x}}\right] + \sqrt{x}\left(4x^3 - 15x^2\right)$

$= \frac{1}{2}x^{7/2} - \frac{5}{2}x^{5/2} + 3x^{-1/2} + 4x^{7/2} - 15x^{5/2}$

Match!

$= \boxed{\frac{9}{2}x^{7/2} - \frac{35}{2}x^{5/2} + 3x^{-1/2}}$

$f''(x) = \boxed{\frac{63}{4}x^{5/2} - \frac{175}{4}x^{3/2} - \frac{3}{2}x^{-3/2}}$

x^{-8}

5. $$f(x) = \frac{\sqrt{x} - \frac{1}{x^8}}{x^7}$$

Quotient Rule First

$$f'(x) = \frac{x^7\left[\frac{1}{2\sqrt{x}} + 8x^{-9}\right] - \left(\sqrt{x} - \frac{1}{x^8}\right)(7x^6)}{(x^7)^2}$$

$$= \frac{\frac{1}{2}x^{13/2} + 8x^{-2} - 7x^{13/2} + 7x^{-2}}{x^{14}}$$

$$= \frac{\frac{-13}{2}x^{13/2} + 15x^{-2}}{x^{14}} = \boxed{\frac{-13}{2}x^{-15/2} + 15x^{-16}}$$

Match!

<u>OR</u> Simplify First

$$f(x) = \frac{\sqrt{x} - x^{-8}}{x^7} = \frac{\sqrt{x}}{x^7} - \frac{x^{-8}}{x^7} = x^{-13/2} - x^{-15}$$

$$f'(x) = \boxed{-\tfrac{13}{2}x^{-15/2} + 15x^{-16}}$$

6. $g(x) = \dfrac{x}{1+3x^2}$

Quotient Rule

(a) $$g'(x) = \frac{(1+3x^2)(1) - x(6x)}{(1+3x^2)^2}$$

$$= \frac{1+3x^2-6x^2}{(1+3x^2)^2} = \boxed{\frac{1-3x^2}{(1+3x^2)^2}}$$

(b) Horizontal Tangent When $g'(x)=0$

$$g'(x) = \frac{1-3x^2}{(1+3x^2)^2} \overset{\text{set}}{=} 0 \Rightarrow 1-3x^2=0$$

$$\Rightarrow 3x^2=1$$

$$\Rightarrow x^2 = 1/3$$

$$\Rightarrow x = \pm\sqrt{\frac{1}{3}} = \boxed{\pm\frac{1}{\sqrt{3}}}$$

7. $f(x) = \dfrac{7x - x^{7/8}}{x^{1/4} - 6x^2}$ Quotient Rule

$$f'(x) = \frac{\left(x^{1/4} - 6x^2\right)\left(7 - \tfrac{7}{8}x^{-1/8}\right) - \left(7x - x^{7/8}\right)\left(\tfrac{1}{4}x^{-3/4} - 12x\right)}{\left(x^{1/4} - 6x^2\right)^2}$$

$$= \frac{7x^{1/4} - \tfrac{7}{8}x^{1/4}x^{-1/8} - 42x^2 + \tfrac{42}{8}x^2x^{-1/8} - \left[\tfrac{7}{4}x^{1/4} - 84x^2 - \tfrac{1}{4}x^{7/8}x^{-3/4} + 12x(x^{7/8})\right]}{\left(x^{1/4} - 6x^2\right)^2}$$

$$= \frac{7x^{1/4} - \tfrac{7}{8}x^{1/8} - 42x^2 + \boxed{\tfrac{21}{4}x^{15/8}} - \tfrac{7}{4}x^{1/4} + 84x^2 + \tfrac{1}{4}x^{1/8} \boxed{- 12x^{15/8}}}{\left(x^{1/4} - 6x^2\right)^2}$$

$$\boxed{= \frac{\tfrac{21}{4}x^{1/4} - \tfrac{5}{8}x^{1/8} + 42x^2 - \tfrac{27}{4}x^{15/8}}{\left(x^{1/4} - 6x^2\right)^2}}$$