

Worksheet 3, Tuesday, September 23, 2014

- Please *show* all of your work and *justify* all of your answers.

1. Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a)
$$\lim_{x \rightarrow 7} \frac{x^2 - 4x - 21}{x^2 - 3x} =$$

(b)
$$\lim_{x \rightarrow 4} \frac{x^2 - 3x - 4}{|x - 4|} =$$

(c)
$$\lim_{x \rightarrow 4} \frac{x^2 - 2x - 8}{x^2 - 5x + 4} =$$

(d)
$$\lim_{x \rightarrow -5} \frac{\frac{1}{1-x} - \frac{1}{6}}{x^2 + 3x - 10} =$$

(e)
$$\lim_{x \rightarrow 3} \frac{x^2 - 12x + 27}{x^2 - 6x + 9} =$$

(f)
$$\lim_{x \rightarrow 3} \frac{x^2 - 12x + 27}{x^2 - 9} =$$

(g)
$$\lim_{x \rightarrow 4} \frac{x + 2}{4 - x} =$$

(h)
$$\lim_{x \rightarrow -4} \frac{x + 2}{x + 4} =$$

(i)
$$\lim_{x \rightarrow 2} \frac{3 - \sqrt{x + 7}}{x^2 - 3x + 2} =$$

(j)
$$\lim_{x \rightarrow 1} \frac{G(x + 2) + x - 8}{G(2x) - 3x^2 - 3x + 2} = \quad \text{where } G(x) = (x - 1)^2 + 3$$

(k)
$$\lim_{x \rightarrow 7} \frac{x - 7}{|7 - x|} =$$

(l)
$$\lim_{x \rightarrow 5} \frac{f(x^2) - 28}{(f(x))^2 - 10x - 14} = \quad \text{where } f(x) = x + 3$$

2. Prove that $\lim_{x \rightarrow 3} 1 - 2x = -5$ using the $\varepsilon - \delta$ definition of the limit.

3. (a) Suppose that $f(x) = \sqrt{x^2 - 5x + 3}$. Compute $f'(x)$ using the **limit definition of the derivative**.

(b) Suppose that $f(x) = \frac{2 - 5x^2}{7 - 3x}$. Compute $f'(x)$ using the **limit definition of the derivative**.

4. Suppose that $f(x) = 5 - 7x + 4x^2 - x^3$.

(a) Write the **equation of the tangent line** to the curve $y = f(x)$ when $x = 1$. ****Use the limit definition of the derivative when computing the derivative.****

(b) Find the x -coordinate(s), if any, where the tangent line to $f(x)$ is horizontal.

5. State the definition for a function $f(x)$ that is continuous at $x = -7$.

6. Suppose that f and g are functions, **and**

- $\lim_{x \rightarrow 7} g(x) = 3$
- $\lim_{x \rightarrow 2} g(x) = 6$
- $f(3) = 2$
- $g(x)$ is continuous at $x = 7$ and $x = 2$.
- $\lim_{x \rightarrow 3} f(x) = 5$.

Evaluate the following quantities and fully **justify** your answers. Do **not** just put down numbers.

(a) $g(7) =$

(b) $g \circ f(3) =$

(c) $f \circ g(7) =$

(d) Is $f(x)$ continuous at $x = 3$? Why or why not?

7. Consider the function defined by

$$f(x) = \begin{cases} \sqrt{x-3} + 1 & \text{if } x > 3 \\ 0 & \text{if } x = 3 \\ (x-2)^2 & \text{if } 1 < x < 3 \\ x + \frac{1}{3} & \text{if } 0 < x \leq 1 \\ \frac{1}{x+3} & \text{if } x < 0 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$.

(b) State the **Domain** of the function $f(x)$.

(c) Compute $\lim_{x \rightarrow -3} f(x) =$

(d) Compute $\lim_{x \rightarrow 0} f(x) =$

(e) Compute $\lim_{x \rightarrow 1} f(x) =$

(f) Compute $\lim_{x \rightarrow 3} f(x) =$

(g) State all the value(s) at which f is discontinuous. Justify your answer(s) using the definition of continuity.