Math 111, Fall 2014

Worksheet 2, Tuesday, September 16, 2014

1. Compute the following limits. Justify your answers. Be clear if they equal a value, or $+\infty$, $-\infty$, or DNE.

   (a) $\displaystyle \lim_{x\to 2} \frac{x^2+6x+8}{x+2}$

   (b) $\displaystyle \lim_{x\to 2} \frac{x^2+6x+8}{x-2}$

   (c) $\displaystyle \lim_{x\to 2} \frac{x^2-6x+8}{x-2}$

   (d) $\displaystyle \lim_{x\to 2} \frac{x^2+5x-14}{x^2-4x+12}$

   (e) $\displaystyle \lim_{x\to 2} \frac{x^2+5x-14}{x^2-8x+12}$

   (f) $\displaystyle \lim_{x\to -3} \frac{x^2+4x+3}{x^2-2x-15}$

   (g) $\displaystyle \lim_{x\to -3} \frac{x^2+4x+3}{x^2+6x+9}$

   (h) $\displaystyle \lim_{t\to 1} \frac{t-1}{g(t^2)-3}$ where $g(t)=2t+1$.

   (i) $\displaystyle \lim_{x\to 0} \frac{x+1}{x(x+2)}$

   (j) $\displaystyle \lim_{x\to 3} \frac{x^2-2x-3}{x^2-6x+9}$

   (k) $\displaystyle \lim_{x\to -5} \frac{\frac{1}{4-x}-\frac{1}{9}}{x+5}$

   (l) $\displaystyle \lim_{x\to -3} \frac{x^2-4x-21}{\sqrt{1-x}-2}$

(m) Let $g(x) = \sqrt{x}$. Compute $\lim\limits_{s \to 1} \dfrac{g(s^2+8) - 3}{s - 1}$

(n) Let $f(x) = \dfrac{1}{x}$. Compute $\lim\limits_{t \to 2} \dfrac{f(t-1) - 2f(t)}{t^2 - 4}$

(o) $\lim\limits_{x \to 4} \dfrac{|x-4|}{x-4}$

(p) $\lim\limits_{x \to -1} \dfrac{1}{|x+1|}$ (Hint: what's the graph of this function?)

2. Write out the rigorous $\varepsilon - \delta$ **Definition of the Limit** $\lim\limits_{x \to a} f(x) = L$. (Use your notes if you need to. You must learn this statement.)
3. Read the following written $\varepsilon - \delta$ example proof.

Prove : $\lim\limits_{x \to 2} 7x - 6 = 8$ using the $\varepsilon - \delta$ **Definition of the Limit**.

Scratchwork: we want $|f(x) - L| = |(7x - 6) - 8| < \varepsilon$; what restrictions on $|x - 2|$ make that possible?

$$|f(x) - L| = |(7x-6) - 8| = |7x - 14| = |7(x-2)| = |7||x-2| = 7|x-2| \text{ (want } < \varepsilon)$$

$$7|x-2| < \varepsilon \text{ means } |x-2| < \frac{\varepsilon}{7}, \qquad \text{so choose } \delta = \frac{\varepsilon}{7}.$$

Proof: Let $\varepsilon > 0$ be given. Choose $\delta = \dfrac{\varepsilon}{7}$. Given x such that $0 < |x-2| < \delta$, then

$$|f(x) - L| = |(7x-6) - 8| = |7x - 14| = |7(x-2)| = |7||x-2| = 7|x-2| < 7 \cdot \frac{\varepsilon}{7} = \varepsilon.$$

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4. Follow the example above to give an $\varepsilon - \delta$ proof that $\lim\limits_{x \to 1} 5x - 3 = 2$.

Turn in your solutions