Math 111, Section 01, Fall 2014 ANSWER KEY!!!

Worksheet 1, Tuesday, September 9th, 2014

1. Simplify each of the following expressions. Show your work.

   We clear the denominator by flipping and multiplying...

   (a) $$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} \cdot \frac{\left(\frac{d}{c}\right)}{\left(\frac{d}{c}\right)} = \frac{\left(\frac{a}{b}\right)\left(\frac{d}{c}\right)}{1} = \boxed{\frac{ad}{bc}}$$

   (b) $$\frac{1}{\left(\frac{a}{b}\right)} = \frac{1}{\left(\frac{a}{b}\right)} \cdot \frac{\left(\frac{b}{a}\right)}{\left(\frac{b}{a}\right)} = \frac{\left(\frac{b}{a}\right)}{1} = \boxed{\frac{b}{a}}$$

   (c) $$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} \cdot \frac{\left(\frac{1}{c}\right)}{\left(\frac{1}{c}\right)} = \frac{\left(\frac{a}{b}\right) \cdot \left(\frac{1}{c}\right)}{1} = \boxed{\frac{a}{bc}}$$

   (d) $$\frac{a}{\left(\frac{b}{c}\right)} = \frac{a}{\left(\frac{b}{c}\right)} \cdot \frac{\left(\frac{c}{b}\right)}{\left(\frac{c}{b}\right)} = \frac{a \cdot \left(\frac{c}{b}\right)}{1} = \boxed{\frac{ac}{b}}$$

2. Solve each of the following equations (if possible):

   (a) $x^2 - 4x - 21 = 0$

   Factor $(x-7)(x+3) = 0$ means either $x - 7 = 0$ or $x + 3 = 0$. Finally, $\boxed{x = 7}$ or $\boxed{x = -3}$.

   (b) $x^2 - x + 7 = 0$

   Try the Quadratic Formula:

   $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Here $a = 1$, $b = -1$ and $c = 7$.

   Then $x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(7)}}{2(1)} = \dfrac{1 \pm \sqrt{1 - 28}}{2} = \dfrac{1 \pm \sqrt{-27}}{2}$

   $\boxed{\text{No Real solution}}$ because we have a negative discriminant ($b^2 - 4ac$).

   (c) $x^2 + 2x - 4 = 0$

   Again, try the Quadratic Formula:

   $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Here $a = 1$, $b = 2$ and $c = -4$.

Then $x = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(-4)}}{2(1)} = \dfrac{-2 \pm \sqrt{4+16}}{2} = \dfrac{-2 \pm \sqrt{20}}{2} = \dfrac{-2 \pm \sqrt{4 \cdot 5}}{2}$

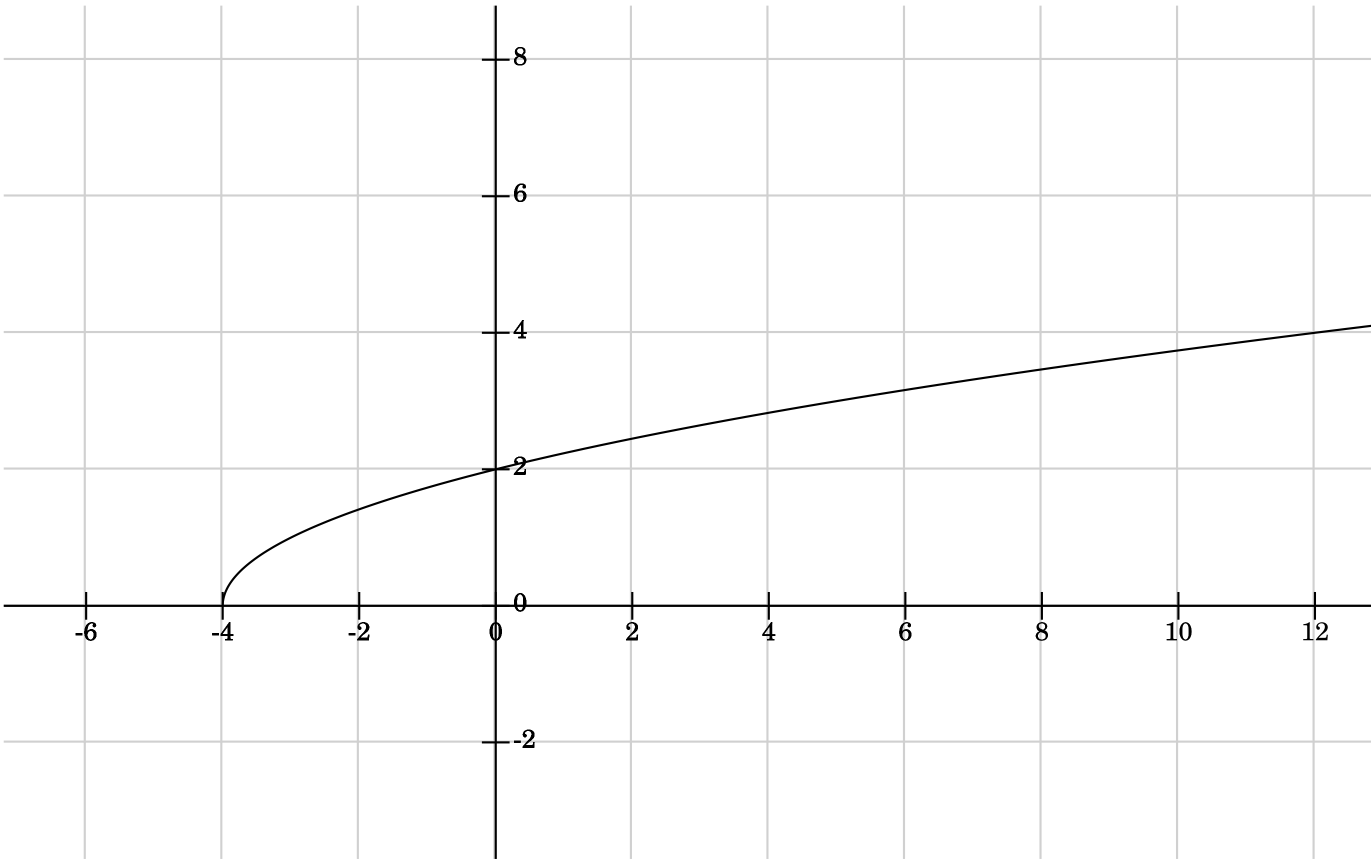
$= \dfrac{-2 \pm \sqrt{4}\sqrt{5}}{2} = \dfrac{-2 \pm 2\sqrt{5}}{2} = \boxed{-1 \pm \sqrt{5}}$

3. YES or NO: Does $\sqrt{x^2+4} = x+2$? Why or why not?
   NO, equal functions must take the same value at *every* point. Here test $x = 1$. $\sqrt{5} \neq 3$.

4. Recall from class that we saw the graphs of $f(x) = \sqrt{x}$ and $g(x) = \dfrac{1}{x}$. Use these graphs to help you do the following:

   (a) Sketch the graph of $F(x) = \sqrt{x+4}$. Discuss the Domain and Range for this new function.

   $\boxed{\text{Domain}=\{x : x \geq -4\} \qquad \text{Range}=[0,\infty) = \{y : y \geq 0\}}$



   (b) Sketch the graph of $G(x) = \dfrac{1}{x-6}$. Discuss the Domain and Range for this new function.

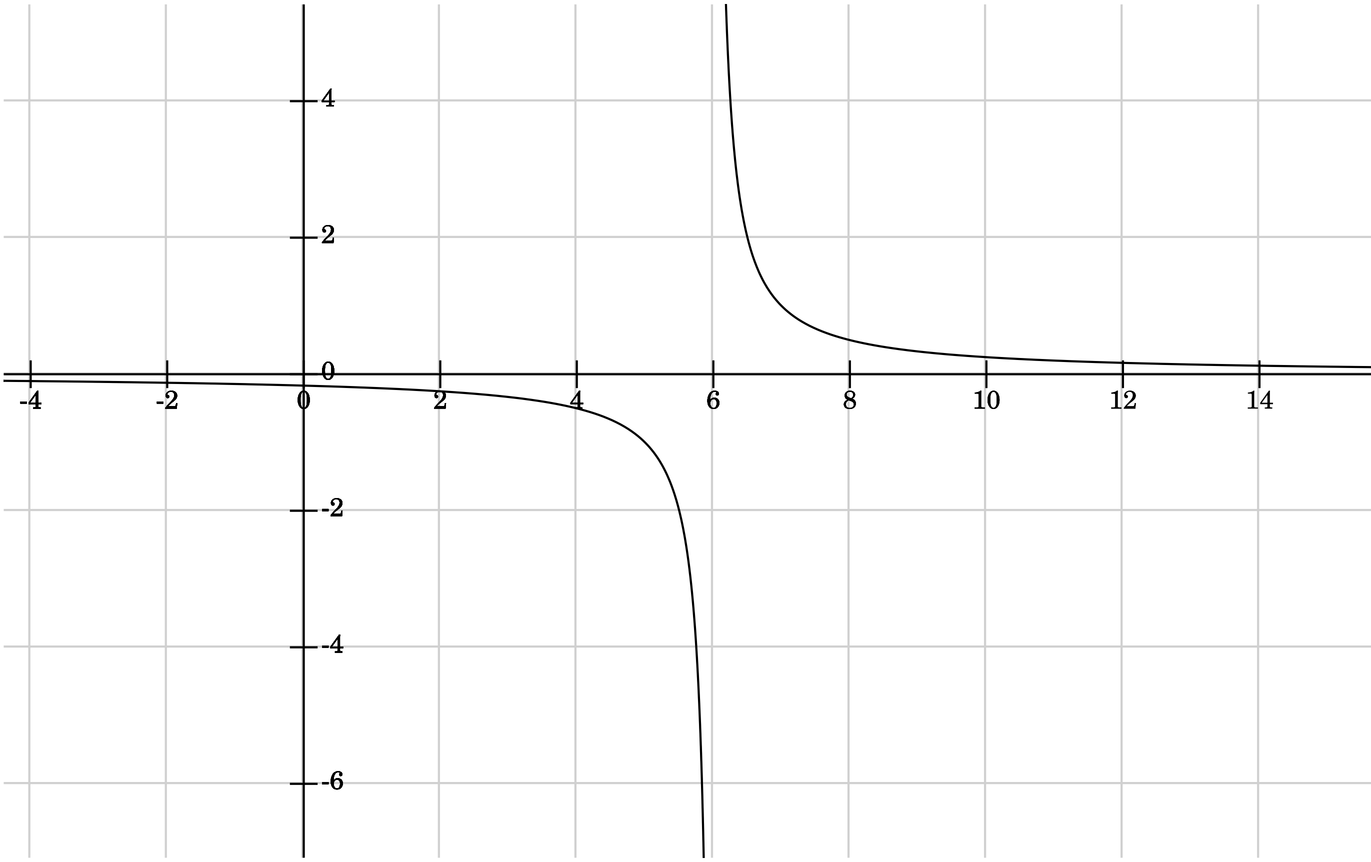
   $\boxed{\text{Domain}=\{x : x \neq 6\} \qquad \text{Range}=(-\infty,0) \cup (0,\infty) = \{y : y \neq 0\}}$

   Use your sketch to answer the following questions:

   (1) Compute RHL: $\displaystyle\lim_{x \to 6^+} \frac{1}{x-6} = +\infty$

(2) Compute LHL: $\lim_{x\to 6^-} \frac{1}{x-6} = -\infty$

(3) What can you decide about the two-sided limit $\lim_{x\to 6} \frac{1}{x-6}$ = DNE because RHL$\neq$LHL

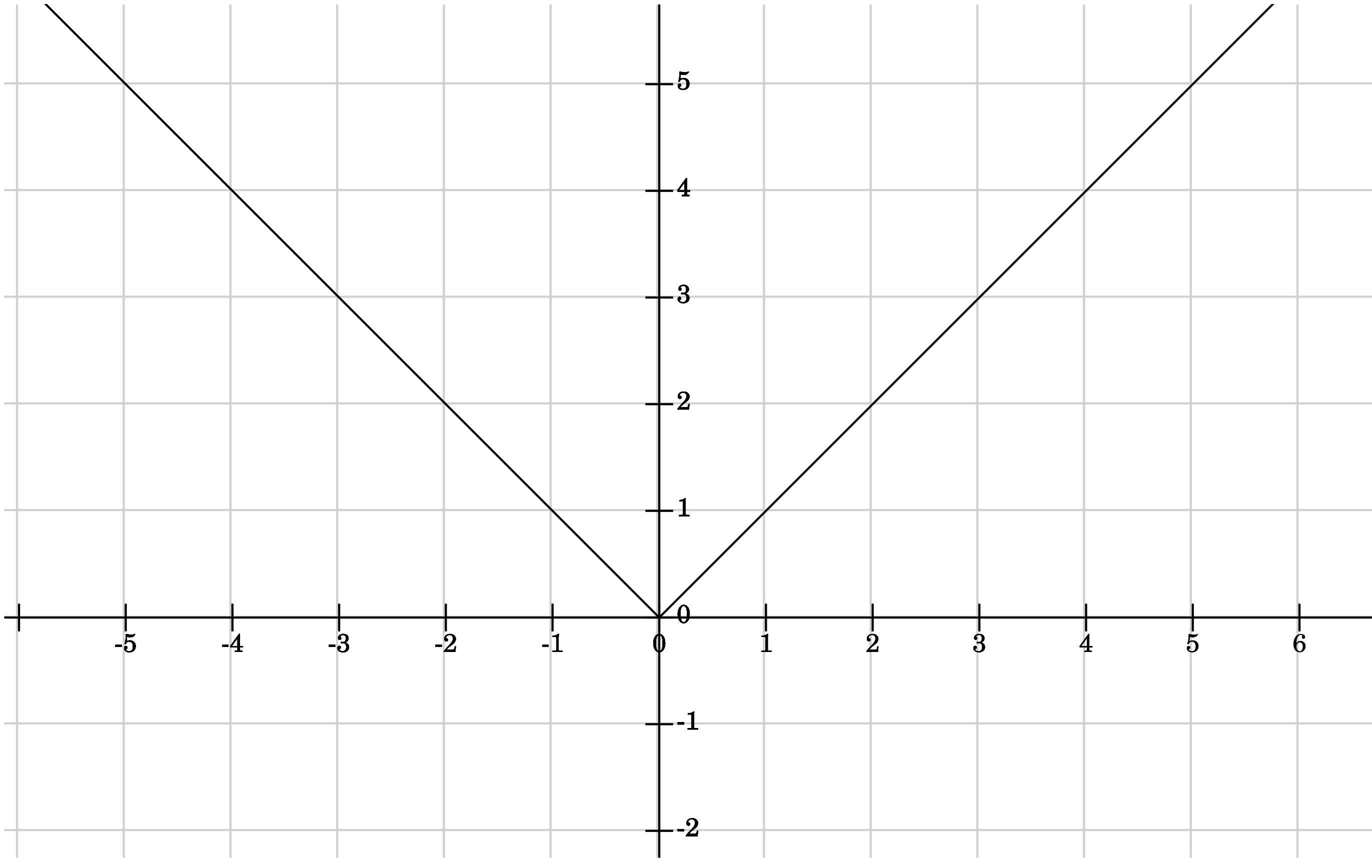


5. The Absolute Value Function $f(x) = |x|$ is a *piece-wise defined function* defined by

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

(a) Give the Domain and Range for this function. Graph the absolute value function.

Domain=$\mathbb{R}$ Range=$\{y : y \geq 0\}$



Now let's similarly write out the piece-wise definition for $f(x) = |x - 3|$

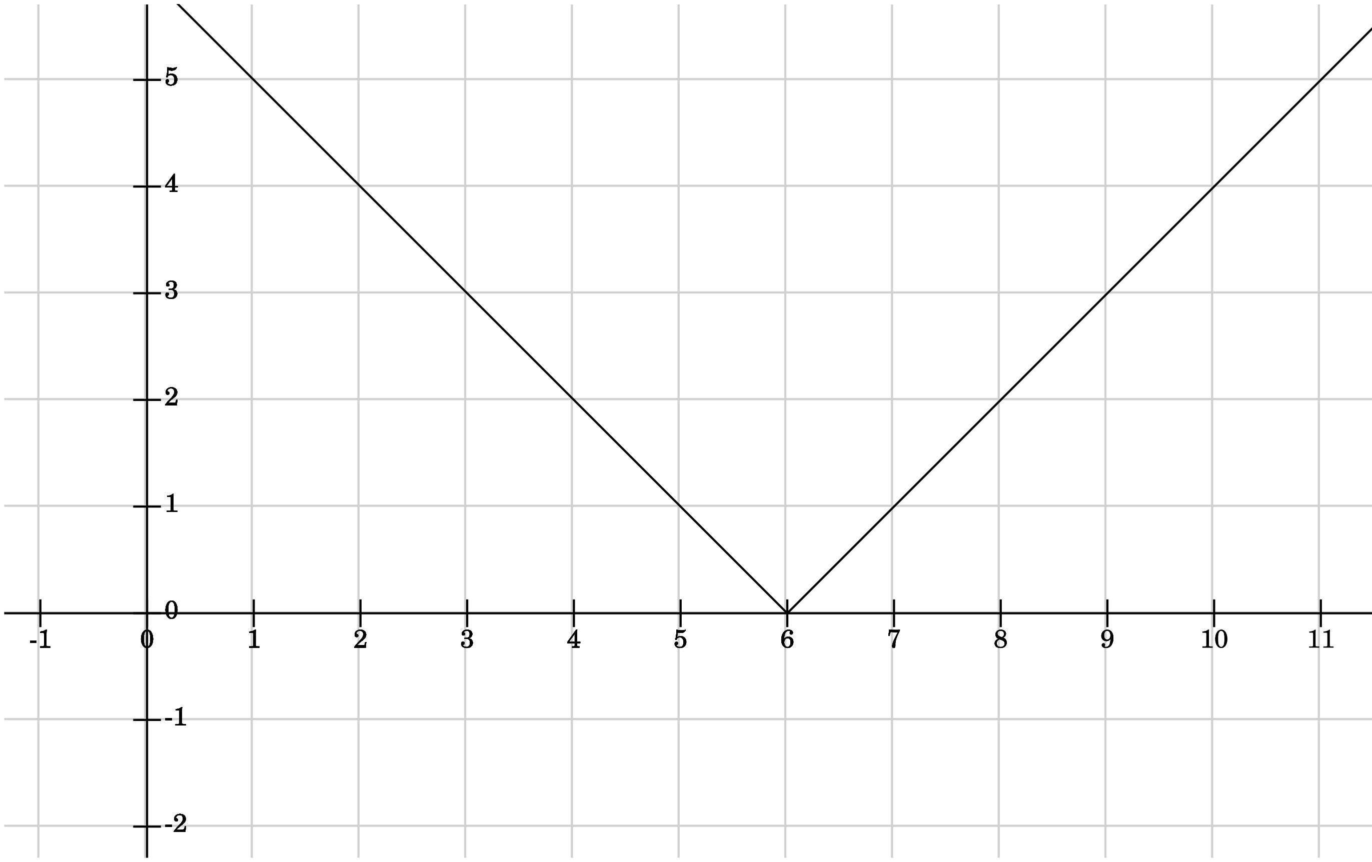
$$f(x) = |x-3| = \begin{cases} x-3 & \text{if } x-3 \geq 0 \\ -(x-3) & \text{if } x-3 < 0 \end{cases} = \begin{cases} x-3 & \text{if } x \geq 3 \\ -x+3 & \text{if } x < 3 \end{cases}$$

(b) Now consider $g(x) = |x - 6|$. Follow the above example for $f(x) = |x - 3|$ to write out the piece-wise defined definition of this function $g(x) = |x - 6|$ carefully. THEN use that definition to graph the function g. Discuss how this graph relates to the graph of $f(x) = |x|$.

$$g(x) = |x-6| = \begin{cases} x-6 & \text{if } x-6 \geq 0 \\ -(x-6) & \text{if } x-6 < 0 \end{cases} = \begin{cases} x-6 & \text{if } x \geq 6 \\ 6-x & \text{if } x < 6 \end{cases}$$

Domain=$\mathbb{R}$ Range=$\{y : y \geq 0\}$

This appears to be a shift of the graph $|x|$ to the right 6 units.

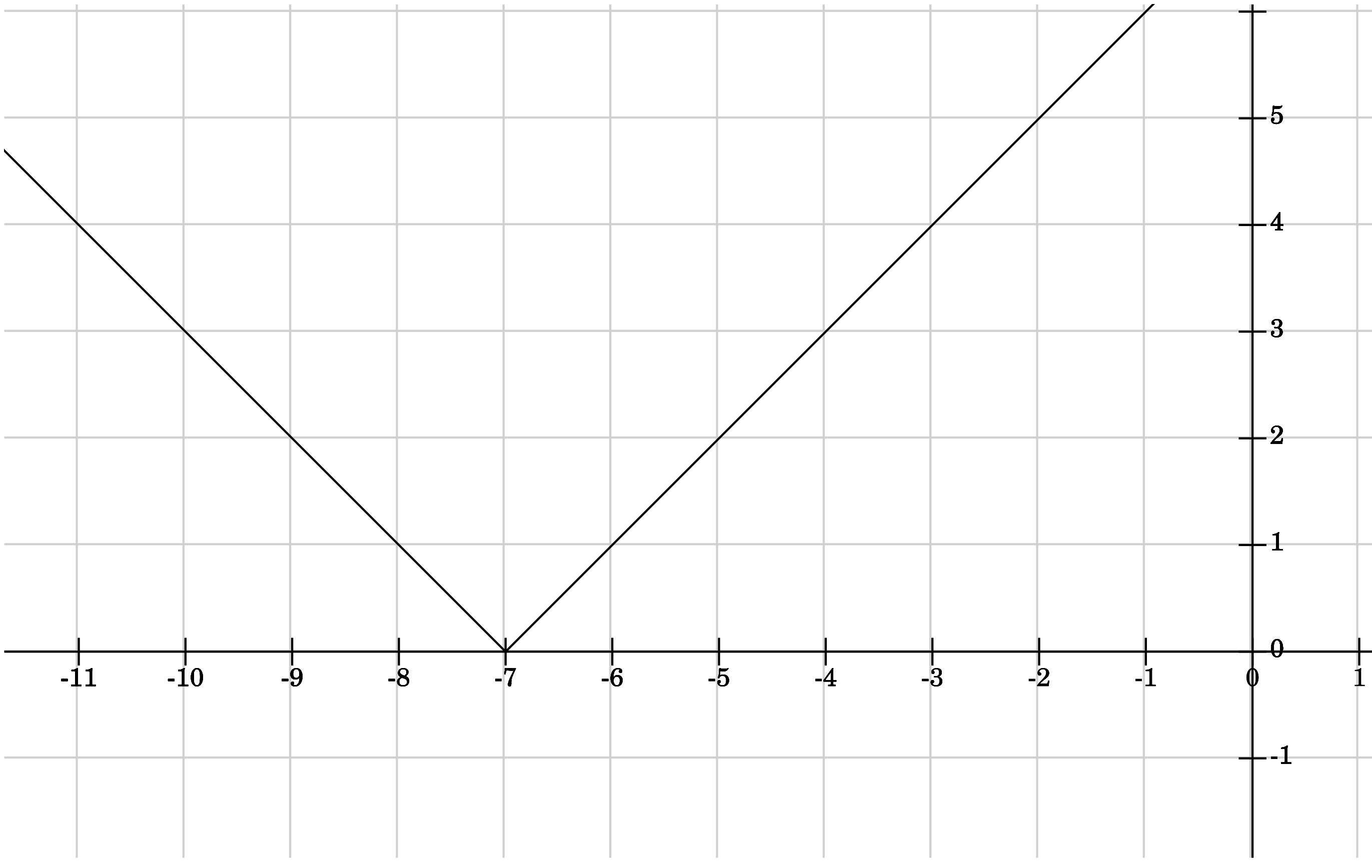


(c) Now consider $h(x) = |x + 7|$. Write out the piece-wise defined definition of this function carefully. THEN use that definition to graph the function h. Discuss how this graph relates to the graph of $f(x) = |x|$.

$$h(x) = |x+7| = \begin{cases} x+7 & \text{if } x+7 \geq 0 \\ -(x+7) & \text{if } x+7 < 0 \end{cases} = \begin{cases} x+7 & \text{if } x \geq -7 \\ -x-7 & \text{if } x < -7 \end{cases}$$

Domain=$\mathbb{R}$ Range=$\{y : y \geq 0\}$

This appears to be a shift of the graph $|x|$ to the left 7 units.



5
4
3
2
1
0
-1
-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1

6. Let $f(x) = \dfrac{x+1}{x-1}$. Compute $f(f(2))$. Compute and simplify $f(f(x))$. Hint: first find a large formula for $f(f(x))$. Then simplify by finding common denominators.

First $f(2) = \dfrac{2+1}{2-1} = \dfrac{3}{1} = 3$. Then $f(f(2)) = f(3) = \dfrac{3+1}{3-1} = \dfrac{4}{2} = \boxed{2}$.

Next, $f(f(x)) = f\left(\dfrac{x+1}{x-1}\right) = \dfrac{\dfrac{x+1}{x-1}+1}{\dfrac{x+1}{x-1}-1} = \dfrac{\dfrac{x+1}{x-1}+\dfrac{x-1}{x-1}}{\dfrac{x+1}{x-1}-\dfrac{x-1}{x-1}}$

$$= \frac{\dfrac{x+1+x-1}{x-1}}{\dfrac{x+1-(x-1)}{x-1}} = \frac{\dfrac{x+1+x-1}{x-1}}{\dfrac{x+1-x+1}{x-1}} = \frac{\dfrac{2x}{x-1}}{\dfrac{2}{x-1}} = \left(\frac{2x}{x-1}\right)\left(\frac{x-1}{2}\right) = \boxed{x}$$

7. Consider the function defined by

$$f(x) = \begin{cases} 2-x & \text{if } x < -1 \\ x+4 & \text{if } -1 < x \leq 1 \\ (x-2)^2 & \text{if } x > 1 \end{cases}$$

Graph $f(x)$ and find its Domain and Range. Discuss the behavior of the function near $x = \pm 1$. That is, use your graph to compute each of the following

(a) $\lim_{x\to 1^+} f(x) = 1$ (b) $\lim_{x\to 1^-} f(x) = 5$ (c) $\lim_{x\to -1^+} f(x) = 3$ (d) $\lim_{x\to -1^-} f(x) = 3$

See me for a sketch.

$\boxed{\text{Domain}=\{x : x \neq -1\} \quad \text{Range}=\{y : y \geq 0\}}$

Don't need to write this, but here is the idea of the one sided limits: It appears that the function is approaching $y = 3$ as the input values approach $x = -1$ from **either** the left or the right. That is from the negative or positive direction. Even though the function is not defined at $x = 3$, the function appears to be approaching $y = 3$.

It appears that the function is approaching $y = 5$ as the input values approach $x = 1$ from the left, and it appears that the function is approaching $y = 1$ as the input values approach $x = 1$ from the left.

8. Simplify each of the following expressions.

(a) $\dfrac{x^2+6x+8}{x^2-4} = \dfrac{(x+2)(x+4)}{(x+2)(x-2)} = \boxed{\dfrac{x+4}{x-2}}$

(b) $\dfrac{x^2+6x+8}{x^2-5x-14} = \dfrac{(x+2)(x+4)}{(x+2)(x-7)} = \boxed{\dfrac{x+4}{x-7}}$

(c) $\dfrac{x^2-6x+8}{x^2-x-2} = \dfrac{(x-2)(x-4)}{(x-2)(x+1)} = \boxed{\dfrac{x-4}{x+1}}$

(d) $\dfrac{1}{t\sqrt{1+t}} - \dfrac{1}{t} = \dfrac{1}{t\sqrt{1+t}} - \dfrac{\sqrt{1+t}}{t\sqrt{1+t}} = \boxed{\dfrac{1-\sqrt{1+t}}{t\sqrt{1+t}}}$

(e) $\dfrac{t-1}{g(t^2)-3}$, where $g(t) = 2t+1$

$$\frac{t-1}{g(t^2)-3} = \frac{t-1}{(2t^2+1)-3} = \frac{t-1}{2t^2-2} = \frac{t-1}{2(t^2-1)} = \frac{t-1}{2(t-1)(t+1)} = \boxed{\frac{1}{2(t+1)}}$$

(f) $\dfrac{x^2-13x+42}{x^2-4x-12} = \dfrac{(x-7)(x-6)}{(x+2)(x-6)} = \boxed{\dfrac{x-7}{x+2}}$

9. Given two functions f and g. The **Composition** of f and g is defined by

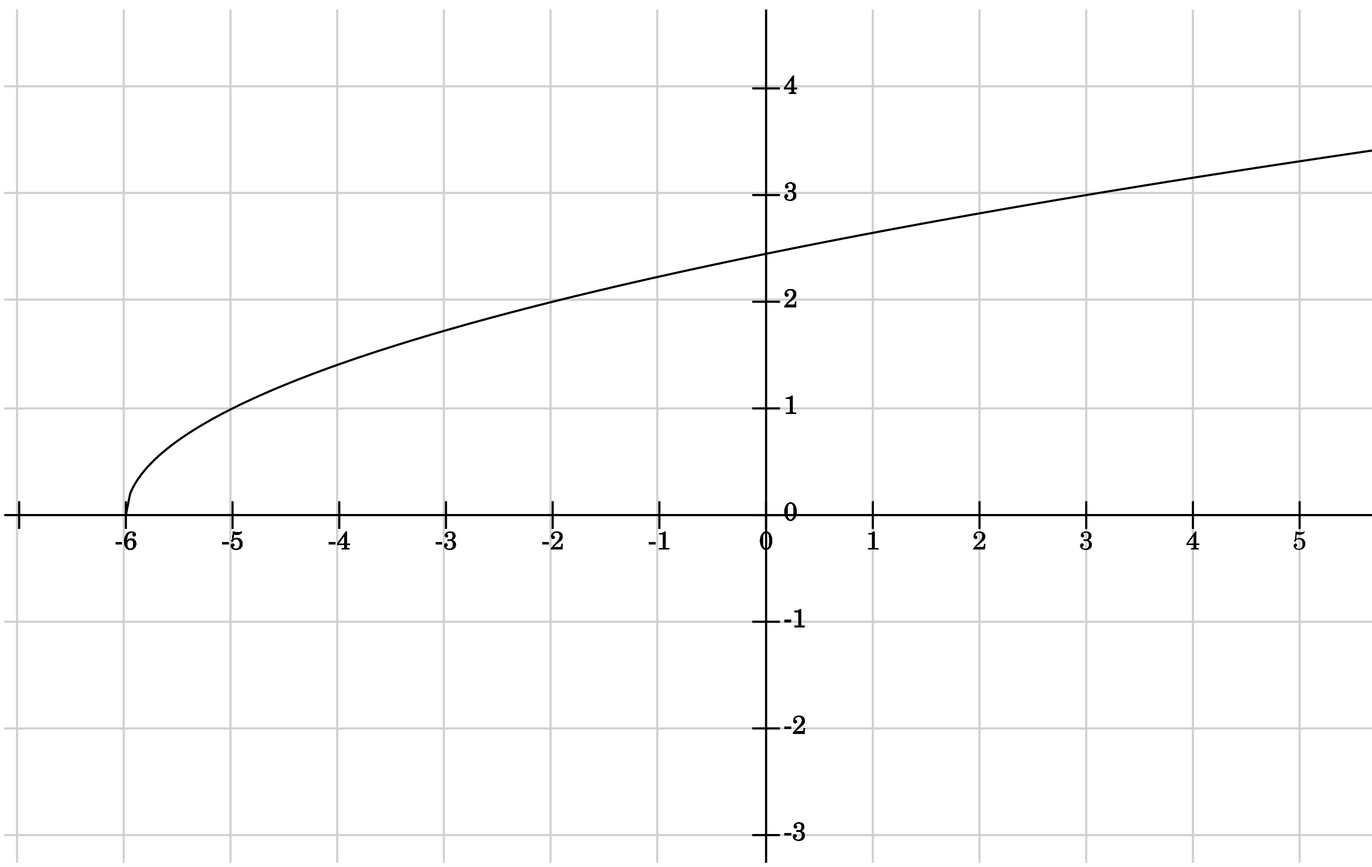
$$f \circ g(x) = f(g(x))$$

(a) Discuss what the Domain of $f \circ g$ is.

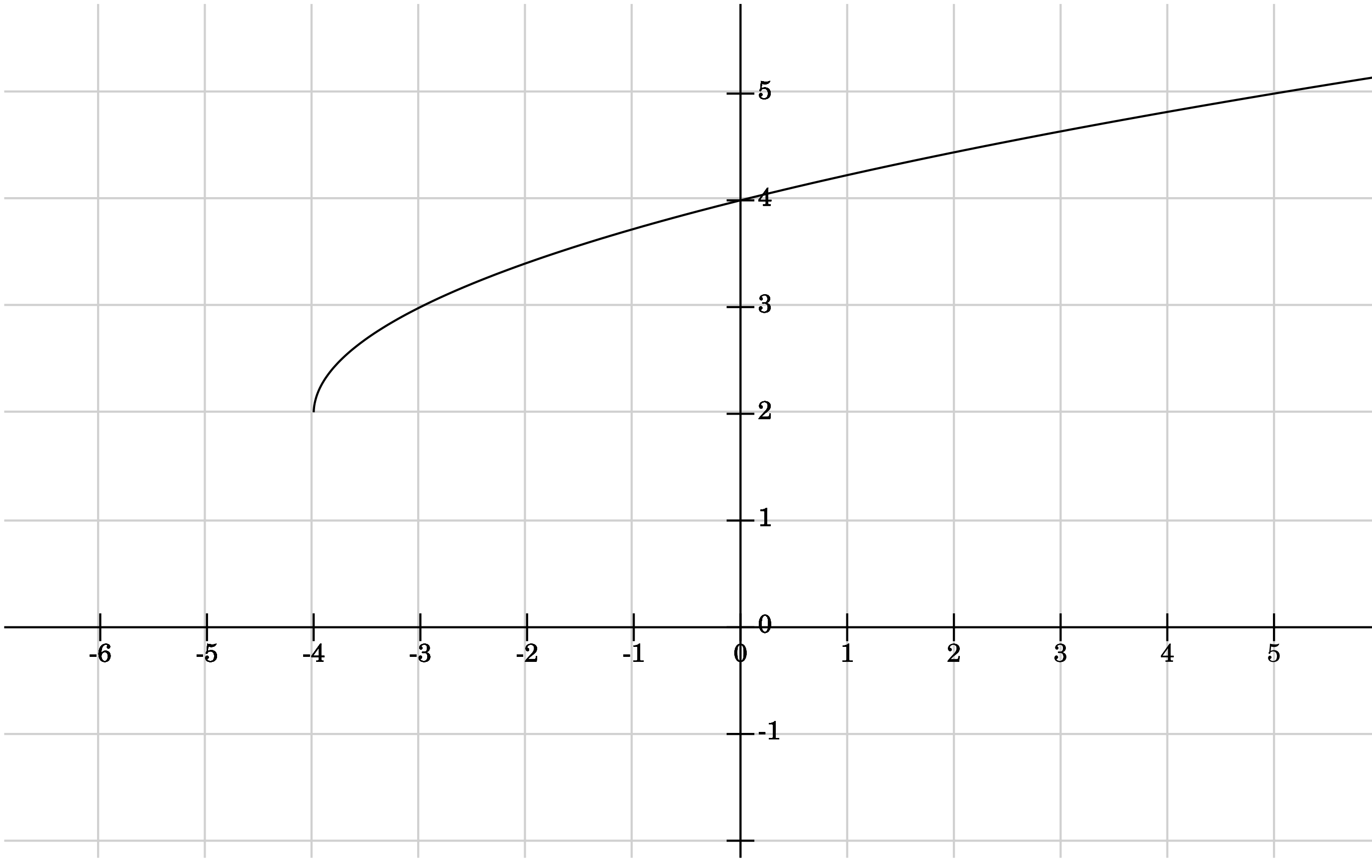
The Domain of the composite function is the set of all x values such that x is in the Domain of g and THEN that output $g(x)$ is in turn in the Domain of f.

(b) Take $f(x) = \sqrt{x+4}$ and $g(x) = x+2$. Compute and graph both $f \circ g$ and $g \circ f$. Discuss whether or not $f \circ g$ equals $g \circ f$. (Hint: what does itmean for two functions to be equal?)

First, $f \circ g(x) = f(g(x)) = f(x+2) = \sqrt{(x+2)+4} = \boxed{\sqrt{x+6}}$.



Second, $g \circ f(x) = g(f(x)) = g(\sqrt{x+4}) = \boxed{\sqrt{x+4}+2}$.



Notice that these are not the same functions. They don't have the same graphs. Equal functions must take the same value at every element of the domain. Notice that they also don't even have the same domains:

Domain $f \circ g = \{x : x \geq -6\}$ and Domain $g \circ f = \{x : x \geq -4\}$.