Math 111, Section 01, Fall 2014

Worksheet 1, Tuesday, September 9th, 2014

1. Simplify each of the following expressions. Show your work.

   (a) $\dfrac{\left(\dfrac{a}{b}\right)}{\left(\dfrac{c}{d}\right)}$ (b) $\dfrac{1}{\left(\dfrac{a}{b}\right)}$

   (c) $\dfrac{\left(\dfrac{a}{b}\right)}{c}$ (d) $\dfrac{a}{\left(\dfrac{b}{c}\right)}$

2. Solve each of the following equations (if possible):

   (a) $x^2 - 4x - 21 = 0$ (b) $x^2 - x + 7 = 0$ (c) $x^2 + 2x - 4 = 0$

3. YES or NO: Does $\sqrt{x^2 + 4} = x + 2$? Why or why not?

   Warning: Square Roots do not split up over sums!

Read the following:

Recall that $\lim_{x \to a^+} f(x) = L$ is the **Right Hand Limit**. We will denote this as **RHL**. It means you are studying the limiting value (if it exists) of the output function values as the input values approach a *from the right of* a.

Also recall that $\lim_{x \to a^-} f(x) = L$ is the **Left Hand Limit**. We will denote this as **LHL**. It means you are studying the limiting value (if it exists) of the output function values as the input values approach a *from the left of* a.

We write $\lim_{x \to a} f(x) = L$ to represent the full **two-sided limit**. We have the following result.

Theorem: $\boxed{\lim_{x \to a} f(x) = L \text{ if and only if } \lim_{x \to a^+} f(x) = L \text{ and } \lim_{x \to a^-} f(x) = L.}$

That is, the two-sided limit exists at a if and only if **both** the one-sided limits, from the right and from the left, exist and are equal. If RHL$\neq$ LHL then we say the two-sided limit Does Not Exist or **DNE**.

4. Recall from class that we saw the graphs of $f(x) = \sqrt{x}$ and $g(x) = \dfrac{1}{x}$. Use these graphs to help you do the following:

(a) Sketch the graph of $F(x) = \sqrt{x+4}$. Discuss the Domain and Range for this new function.

(b) Sketch the graph of $G(x) = \dfrac{1}{x-6}$. Discuss the Domain and Range for this new function.

Use your sketch to answer the following questions:

(1) Compute RHL: $\displaystyle\lim_{x\to 6^+} \frac{1}{x-6} =$

(2) Compute LHL: $\displaystyle\lim_{x\to 6^-} \frac{1}{x-6} =$

(3) What can you decide about the two-sided limit $\displaystyle\lim_{x\to 6} \frac{1}{x-6}$ =?

5. The Absolute Value Function $f(x) = |x|$ is a *piece-wise defined function* defined by

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

(a) Give the Domain and Range for this function. Graph the absolute value function.

Now let's similarly write out the piece-wise definition for $f(x) = |x-3|$

$$f(x) = |x-3| = \begin{cases} x-3 & \text{if } x-3 \geq 0 \\ -(x-3) & \text{if } x-3 < 0 \end{cases} = \begin{cases} x-3 & \text{if } x \geq 3 \\ -x+3 & \text{if } x < 3 \end{cases}$$

(b) Now consider $g(x) = |x-6|$. Follow the above example for $f(x) = |x-3|$ to write out the piece-wise defined definition of this function $g(x) = |x-6|$ carefully. THEN use that definition to graph the function g. Discuss how this graph relates to the graph of $f(x) = |x|$.

(c) Now consider $h(x) = |x+7|$. Write out the piece-wise defined definition of this function carefully. THEN use that definition to graph the function h. Discuss how this graph relates to the graph of $f(x) = |x|$.

6. Let $f(x) = \dfrac{x+1}{x-1}$. Compute $f(f(2))$. Compute and simplify $f(f(x))$. Hint: first find a large formula for $f(f(x))$. Then simplify by finding common denominators.

7. Consider the function defined piece-wise by

$$f(x) = \begin{cases} 2-x & \text{if } x < -1 \\ x+4 & \text{if } -1 < x \leq 1 \\ (x-2)^2 & \text{if } x > 1 \end{cases}$$

Graph $f(x)$ and find its Domain and Range. Discuss the behavior of the function near $x = \pm 1$. That is, use your graph to compute each of the following

(a) $\lim_{x \to 1^+} f(x) =$ (b) $\lim_{x \to 1^-} f(x) =$ (c) $\lim_{x \to -1^+} f(x) =$ (d) $\lim_{x \to -1^-} f(x) =$

8. Simplify each of the following expressions.

(a) $\dfrac{x^2+6x+8}{x^2-4}$

(b) $\dfrac{x^2+6x+8}{x^2-5x-14}$

(c) $\dfrac{x^2-6x+8}{x^2-x-2}$

(d) $\dfrac{1}{t\sqrt{1+t}} - \dfrac{1}{t}$

(e) $\dfrac{t-1}{g(t^2)-3}$, where $g(t) = 2t+1$

(f) $\dfrac{x^2-13x+42}{x^2-4x-12}$

9. Given two functions f and g. The **Composition** of f and g is defined by

$$f \circ g(x) = f(g(x))$$

(a) Discuss what the Domain of $f \circ g$ is.

(b) Take $f(x) = \sqrt{x+4}$ and $g(x) = x+2$. Compute and graph both $f \circ g$ and $g \circ f$. Discuss whether or not $f \circ g$ equals $g \circ f$. (Hint: what does it mean for two functions to be equal?)

Turn in your own solutions.

You do need to understand ALL of these problems.

I will post answer keys on the class webpage.