Math 111, D. Benedetto

**What you need to know for Exam 1**

You should know everything from the beginning of the course up to (and including) Section 2.2. The following is a list of most of the topics covered. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.** Remember, no calculators in any exams.

- 1.1: What is a function? (Vertical line test.) Domain of a function. Piecewise-defined functions, including $|x|$.
- 1.2: Standard functions: polynomials (including constant and linear), power functions (including root functions and things like $1/x$), rational functions.
- 1.3: Adding, subtracting, multiplying, and dividing functions. Composition of functions. Given the graph of $y = f(x)$, what do $y = f(x) \pm c$, $y = f(x \pm c)$, $y = cf(x)$, $y = f(cx)$, $y = -f(x)$, $y = f(-x)$, and $y = |f(x)|$ look like? (See boxes on pp. 36–37, and Example 5.)
- 1.5: Limits. Intuitive idea of what a limit is. Estimating limits using tables and computations. Be familiar with the various ways a limit can fail to exist. Right- and left-hand limits; and Box 3, page 55. Infinite limits. Vertical asymptotes.
- 1.6: Limit laws: sum, difference, constant multiple, product, quotient, power, root laws. Computing limits of polynomials, rational functions, and other functions. Computing harder limits, like $0/0$, using algebraic tricks.
- 1.7: The rigorous (that is, $\varepsilon$-$\delta$) definition of a limit. Try to have at least some intuitive understanding of why the $\varepsilon$-$\delta$ definition agrees with the idea of a limit from Section 1.5. Also, be able to prove limits for linear polynomials using $\varepsilon$-$\delta$.
- 1.8: Continuity: at a number $a$, or on an interval $I$. Know both the official definition ($\lim_{x\to a} f(x) = f(a)$) and the intuitive idea (you can draw it without lifting your pencil off the paper). The three ways continuity can fail ($f(a)$ not defined; or $\lim_{x\to a} f(x)$ not defined; or both defined, but not equal). Continuity from the right or left. Theorems 4–9 (for continuity of sums, differences, products, and quotients, not to mention polynomials, rational functions, trig functions, and root functions, as well as compositions).
- 2.1: Limit definition of derivatives. (We've stuck to box 4 on page 107; just don't mix-and-match parts of one with the other.) Computing derivatives with the limit definition. Derivatives as slopes of tangent lines or instantaneous velocities or rates of change. All derivatives for this exam will be computed using the limit definition of the derivative. What is it?
- 2.2: The derivative as a function. Differentiability. How can differentiability fail? Differentiable implies continuous, but not necessarily the other way around. The graph of $f'$ in relation to the graph of $f$. Higher derivatives.

## Some Things You **Don't** Need to Know For This Exam

- 1.1: Symmetry. Increasing and Decreasing (for right now anyhow).
- 1.2: Algebraic functions. (That is, you don't need to know the definition, though you **do** need to be able to deal with functions like $f(x) = \sqrt{x^2 - 3}$.)
- 1.2: Exponential and logarithmic functions (for right now anyhow).
- 1.6: The identifying numbers of all the limit laws.
- 1.6: The greatest integer function.
- 1.6: Boxes 2 or 3, page 68.
- 1.7: Rigorous definitions of strictly right-hand, left-hand, and infinite limits.
- 1.8: The Intermediate Value Theorem (for right now anyhow).

### Tips

- You will be allowed to do the problems in any order, but **make sure you don't accidentally skip any** if you like to jump around.
- Make sure you know how to manipulate functions. If any of the problems from Sections 1.1–1.3 gave you serious trouble, make sure you get things cleared up.
- On $\varepsilon$-$\delta$ proofs, make sure your scratchwork is clearly set aside and marked as scratch. Then, make sure the proof itself is formal and clear. See the handout with on $\varepsilon$-$\delta$ proofs for more tips.

  I won't give you anything worse than a linear polynomial on an $\varepsilon$-$\delta$ problem. So don't panic about things like Exercise 37 (the $\sqrt{x}$ problem).
- You will need to compute some limits using limit laws. Be ready. Know the algebra tricks you need for 0/0 limits. (See the course overview handout from week 1 for a list) Know when to use LHL and RHL sign analysis. Also know how to examine the two cases for absolute value pieces of your functions.
- Make sure you are clear when making a statement about discontinuity. You must tell me which of the three pieces of the definition fail(s).
- You will have to compute some derivatives **from the limit definition**. Be ready to do that. You will get no credit for producing the correct answer by a method (like the "power rule") covered in Section 2.3 or beyond. However, knowing quick ways to do derivatives can aid as a personal double check for you of whether you did the derivative correctly the long way.
- Justify all of your work!! Don't just use random words to explain answers; I will look for clear mathematical statements.