Math 11 Final Review Packet for Remaining Material since Exam #3

Integration Compute each of the following integrals:

1. $\int x(x^2+1)^{14}\,dx = \frac{1}{2}\int u^{14}\,du = \frac{1}{2}\frac{u^{15}}{15} + C = \boxed{\frac{1}{30}(x^2+1)^{15} + C}$

   Here $\begin{cases} u &= x^2+1 \\ du &= 2xdx \\ \frac{1}{2}du &= xdx \end{cases}$

2. $\int \sin(4x)\cos(4x)\,dx = \frac{1}{4}\int u\,du = \frac{1}{4}\frac{u^2}{2} + C = \boxed{\frac{1}{8}\sin^2(4x) + C}$

   Here $\begin{cases} u &= \sin(4x) \\ du &= \cos(4x)4dx \\ \frac{1}{4}du &= \cos(4x)dx \end{cases}$

3. $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|e^x + e^{-x}| + C}$

   Here $\begin{cases} u &= e^x + e^{-x} \\ du &= e^x - e^{-x}dx \end{cases}$

4. $\int \frac{1}{x}\sqrt{1+\ln x}\,dx = \int \sqrt{u}\,du = \frac{2}{3}u^{\frac{3}{2}} + C = \boxed{\frac{2}{3}(1+\ln x)^{\frac{3}{2}} + C}$

   Here $\begin{cases} u &= 1+\ln x \\ du &= \frac{1}{x}dx \end{cases}$

5. $\int \frac{1}{(x+1)\ln(x+1)}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|\ln(x+1)| + C}$

   Here $\begin{cases} u &= \ln(x+1) \\ du &= \frac{1}{x+1}dx \end{cases}$

6. $\int \frac{\sin x}{7+\cos x}\,dx = -\int \frac{1}{u}\,du = -\ln|u| + C = \boxed{-\ln|7+\cos x| + C}$

   Here $\begin{cases} u &= 7+\cos x \\ du &= -\sin x dx \\ -du &= \sin x dx \end{cases}$

7. $\int \frac{6e^x}{e^x+7}\,dx = 6\int \frac{1}{u}\,dx = 6\ln|u| + C = \boxed{6\ln|e^x+7| + C}$

   Here $\begin{cases} u &= e^x+7 \\ du &= e^x dx \end{cases}$

8. $\int \frac{e^{\ln(\sin x)}}{e^{\ln(\cos x+7)}}\,dx = \int \frac{\sin x}{\cos x + 7}\,dx$ and this reduces to #6.

   Or

$$\int \frac{e^{\ln(\sin x)}}{e^{\ln(\cos x+7)}}\,dx = \int e^{\ln(\sin x) - \ln(\cos x+7)}\,dx = \int e^{\ln\left(\frac{\sin x}{\cos x+7}\right)}\,dx$$

$= \int \frac{\sin x}{\cos x + 7}\,dx$ which still reduces to #6.

9. $\int \ln(e^{x^2}e^x e^7)\,dx = \int \ln e^{x^2+x+7}\,dx = \int x^2+x+7\,dx = \boxed{\frac{x^3}{3}+\frac{x^2}{2}+7x+C}$

Or

$\int \ln e^{x^2} + \ln e^x + \ln e^7\,dx = \int x^2+x+7\,dx = \frac{x^3}{3}+\frac{x^2}{2}+7x+C$

10. $\int \frac{6x+3}{x^2+x-5}\,dx = 3\int \frac{1}{u}\,du = 3\ln|u|+C = \boxed{3\ln|x^2+x-5|+C}$

Here $\begin{cases} u &= x^2+x-5 \\ du &= 2x+1dx \end{cases}$

11. $\int \frac{1}{1-2x}\,dx = -\frac{1}{2}\int \frac{1}{u}\,du = -\frac{1}{2}\ln|u|+C = \boxed{-\frac{1}{2}\ln|1-2x|+C}$

Here $\begin{cases} u &= 1-2x \\ du &= -2dx \\ -\frac{1}{2}du &= dx \end{cases}$

12. $\int e^{3x+1}\,dx = \frac{1}{3}\int e^u\,du = \frac{1}{3}e^u + C = \boxed{\frac{1}{3}e^{3x+1}+C}$

Here $\begin{cases} u &= 3x+1 \\ du &= 3dx \\ \frac{1}{3}du &= dx \end{cases}$

13. $\int \frac{e^{-\frac{1}{x^7}}}{x^8}\,dx = \frac{1}{7}\int e^u\,du = \frac{1}{7}e^u + C = \boxed{\frac{1}{7}e^{-\frac{1}{x^7}}+C}$

Here $\begin{cases} u &= -\frac{1}{x^7} \\ du &= 7x^{-8}dx \\ \frac{1}{7}du &= \frac{1}{x^8}dx \end{cases}$

14. $\int \frac{1}{e^x}\,dx = \int e^{-x}\,dx = -\int e^u\,du = -e^u + C = \boxed{-e^{-x}+C}$

Here $\begin{cases} u &= -x \\ du &= -dx \\ -du &= dx \end{cases}$

15. $\int_0^1 \frac{1}{7x+1}\,dx = \frac{1}{7}\int_{u=1}^{u=8} \frac{1}{u}\,du = \frac{1}{7}\ln|u|\Big|_1^8 = \frac{1}{7}(\ln 8 - \ln 1) = \boxed{\frac{1}{7}\ln 8}$

Here $\begin{cases} u &= 7x+1 \\ du &= 7dx \\ \frac{1}{7}du &= dx \end{cases}$ and $\begin{cases} x=0 & \Longrightarrow u=1 \\ x=1 & \Longrightarrow u=8 \end{cases}$

16. $\displaystyle\int_e^{e^2} \frac{1}{x(\ln x)^2}\, dx = \int_{u=1}^{u=2} \frac{1}{u^2}\, du = \left.\frac{u^{-1}}{-1}\right|_1^2 = -\left.\frac{1}{u}\right|_1^2 = -\frac{1}{2} - (-1) = \boxed{\frac{1}{2}}$

Here $\begin{cases} u &= \ln x \\ du &= \frac{1}{x} dx \end{cases}$ and $\begin{cases} x = e & \Longrightarrow u = \ln e = 1 \\ x = e^2 & \Longrightarrow u = \ln e^2 = 2 \end{cases}$

17. $\displaystyle\int_{\ln 4}^{\ln 7} 9e^{2x}\, dx = \frac{9}{2}\int_{\ln 16}^{\ln 49} e^u\, du = \left.\frac{9}{2}e^u\right|_{\ln 16}^{\ln 49} = \frac{9}{2}\left(e^{\ln 49} - e^{\ln 16}\right) = \frac{9}{2}(49-16) = \frac{9}{2}(33) = \boxed{\frac{297}{2}}$

Here $\begin{cases} u &= 2x \\ du &= 2dx \\ \frac{1}{2}du &= dx \end{cases}$ and $\begin{cases} x = \ln 4 & \Longrightarrow u = 2\ln 4 = \ln 4^2 = \ln 16 \\ x = \ln 7 & \Longrightarrow u = 2\ln 7 = \ln 7^2 = \ln 49 \end{cases}$

18. $\displaystyle\int_0^{\ln 3} \left(2 + \frac{1}{e^x}\right)^2 dx = \int_0^{\ln 3} 4 + \frac{4}{e^x} + e^{-2x}\, dx = 4x - 4e^{-x} - \left.\frac{1}{2}e^{-2x}\right|_0^{\ln 3} = (4\ln 3 - 4e^{-\ln 3} - \frac{1}{2}e^{-2\ln 3}) - (0 - 4e^0 - \frac{1}{2}e^0) = 4\ln 3 - 4e^{\ln 3^{-1}} - \frac{1}{2}e^{\ln 3^{-2}} + \frac{9}{2} = 4\ln 3 - (4)3^{-1} - \frac{1}{2}3^{-2} + \frac{9}{2} = 4\ln 3 - (4)\frac{1}{3} - \frac{1}{2}\frac{1}{3^2} + \frac{9}{2} = 4\ln 3 - \frac{4}{3} - \frac{1}{18} + \frac{9}{2} = 4\ln 3 - \frac{24}{18} - \frac{1}{18} + \frac{81}{18} = \boxed{4\ln 3 - \frac{56}{18}}$

19. $\displaystyle\int \frac{we^{w^2}}{17 + e^{w^2}}\, dw = \frac{1}{2}\int \frac{1}{u}\, du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|17 + e^{w^2}| + C = \boxed{\frac{1}{2}\ln\left(17 + e^{w^2}\right) + C}$

Here $\begin{cases} u &= 17 + e^{w^2} \\ du &= e^{w^2}(2w)\, dw \\ \frac{1}{2}du &= we^{w^2}\, dw \end{cases}$

20. $\displaystyle\int_{\ln 2}^{\ln 3} e^{2x}\, dx = \left.\frac{1}{2}e^{2x}\right|_{\ln 2}^{\ln 3} = \frac{1}{2}\left(e^{2\ln 3} - e^{2\ln 2}\right) = \frac{1}{2}\left(e^{\ln(3^2)} - e^{\ln(2^2)}\right)$

$\displaystyle = \frac{1}{2}\left(e^{\ln 9} - e^{\ln 4}\right) = \frac{1}{2}(9-4) = \boxed{\frac{5}{2}}$

OR we can use u-substitution to finish the integral.

$\displaystyle\int_{\ln 2}^{\ln 3} e^{2x}\, dx = \frac{1}{2}\int_{\ln 4}^{\ln 9} e^u\, du = \left.\frac{1}{2}e^u\right|_{\ln 4}^{\ln 9} = \frac{1}{2}\left(e^{\ln 9} - e^{\ln 4}\right) = \frac{9-4}{2} = \boxed{\frac{5}{2}}$

Here $\begin{cases} u &= 2x \\ du &= 2\, dx \\ \frac{1}{2}du &= dx \end{cases}$ and $\begin{cases} x = \ln 2 & \Longrightarrow u = 2\ln 2 = \ln 2^2 = \ln 4 \\ x = \ln 3 & \Longrightarrow u = 2\ln 3 = \ln 3^2 = \ln 9 \end{cases}$

21. $\displaystyle\int \frac{e^{-x}\ln(1+e^{-x})}{1+e^{-x}}\, dx = -\int u\, du = -\frac{u^2}{2} + C = \boxed{-\frac{\left(\ln(1+e^{-x})\right)^2}{2} + C}$

Here $\begin{cases} u &= \ln(1+e^{-x}) \\ du &= \dfrac{1}{1+e^{-x}}(e^{-x})(-1)\, dx \\ -du &= \dfrac{e^{-x}}{1+e^{-x}}\, dx \end{cases}$

22. $\displaystyle\int_e^{e^4} \frac{1}{x\sqrt{\ln x}}\, dx = \int_{u=1}^{u=4} \frac{1}{\sqrt{u}}\, du = \int_{u=1}^{u=4} u^{-\frac{1}{2}}\, du = \left.2u^{\frac{1}{2}}\right|_{u=1}^{u=4} = 2\sqrt{4} - 2\sqrt{1} = \boxed{2}$

Here $\begin{cases} u &= \ln x \\ du &= \dfrac{1}{x}\,dx \end{cases}$ and $\begin{cases} x = e & \Longrightarrow u = \ln e = 1 \\ x = e^4 & \Longrightarrow u = \ln(e^4) = 4 \end{cases}$

23. $\displaystyle\int (e^{3x} + e^{-7x})^2\,dx = \int e^{6x} + 2e^{-4x} + e^{-14x}\,dx = \boxed{\frac{1}{6}e^{6x} - \frac{1}{2}e^{-4x} - \frac{1}{14}e^{-14x} + C}$

24. $\displaystyle\int \frac{1}{2x-1}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \boxed{\frac{1}{2}\ln|2x-1| + C}$

Here $\begin{cases} u &= 2x - 1 \\ du &= 2\,dx \\ \dfrac{1}{2}du &= dx \end{cases}$

Derivatives/Tangent Lines

25. Find the equation of the tangent line to the curve $y = (x+2)e^{-x}$ at the point $(0, 2)$.
$y' = (x+2)e^{-x}(-1) + e^{-x}(1) = e^{-x}(1 - (x+2)) = e^{-x}(1 - x - 2) = e^{-x}(-1 - x)$
Then, the slope at $x = 0$ is given by $y'(0) = e^0(-1 - 0) = -1$.
As a result, the equation of the tangent line is $y - 2 = (-1)(x - 0)$ or $\boxed{y = -x + 2}$.

26. Find the equation of the tangent line to the curve $y = \ln(xe^{-3x})$ at the point $(1, -3)$.
First we can simplify the function $y = \ln(xe^{-3x}) = \ln x + \ln e^{-3x} = \ln x - 3x$.
Then $y' = \dfrac{1}{x} - 3$.
The slope at $x = 1$ is given by $y'(1) = 1 - 3 = -2$.
As a result, the equation of the tangent line is $y + 3 = -2(x - 1)$ or $\boxed{y = -2x - 1}$.

27. Let $y = \dfrac{\ln x}{1 + x^2}$, find $f'(1)$.
Using the Quotient Rule, $y' = \dfrac{(1+x^2)\left(\frac{1}{x}\right) - \ln x(2x)}{(1+x^2)^2} = \dfrac{\frac{1}{x} + x - 2x\ln x}{(1+x^2)^2}$.
Then the slope at $x = 1$ is given by $y'(1) = \dfrac{1 + 1 - 2\ln 1}{4} = \boxed{\dfrac{1}{2}}$.

28. Let $f(x) = x\ln x$ with $x > 0$. Where is $f(x)$ concave up?
First $f'(x) = x\dfrac{1}{x} + \ln x(1) = 1 + \ln x$.
Second $f''(x) = \dfrac{1}{x}$. Notice that the second derivative is always positive for $x > 0$. So f is concave up for all $x > 0$ which is the entire domain of the original function f.

29. Let $x^2e^y = \ln(xy)$. Find $\dfrac{dy}{dx}$.

Implicit differentiation yields $\dfrac{d}{dx}\left(x^2e^y\right) = \dfrac{d}{dx}(\ln(xy))$.

Then $x^2e^y\dfrac{dy}{dx} + e^y 2x = \dfrac{1}{xy}\left(x\dfrac{dy}{dx} + y\right) = \dfrac{1}{y}\dfrac{dy}{dx} + \dfrac{1}{x}$

Simplifying yields $x^2e^y\dfrac{dy}{dx} - \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x} - 2xe^y$

Finally, $\boxed{\dfrac{dy}{dx} = \dfrac{\frac{1}{x} - 2xe^y}{x^2e^y - \frac{1}{y}}}$

30. Find all local maximum and minimum value(s) of the function $f(x) = (x^2 - 7)e^{-x}$.

Compute $f'(x) = (x^2-7)e^{-x}(-1) + e^{-x}(2x) = e^{-x}(-x^2+7+2x)$. Setting $f'(x) = 0$, we need to solve $-x^2 + 2x + 7 = 0$ which has solutions $x = 1 \pm 2\sqrt{2}$. Sign-testing around these points yields a local max at $(1+2\sqrt{2}, f(1+2\sqrt{2})) = (1+2\sqrt{2}, e^{-(1+2\sqrt{2})}(2+4\sqrt{2}))$ and a local min at $(1-2\sqrt{2}, f(1-2\sqrt{2})) = (1-2\sqrt{2}, e^{-(1-2\sqrt{2})}(2-4\sqrt{2}))$.

(Recall that $e^{-x} \neq 0$)

31. Compute the derivatives of the following functions. (Hint: You may want to simplify first.)

(a) $f(x) = \ln(5xe^{-5x}) = \ln(5x) + \ln(e^{-5x}) = \ln(5x) - 5x$

Then $f'(x) = \dfrac{1}{5x}(5) - 5 = \boxed{\dfrac{1}{x} - 5}$

(b) $f(x) = e^{\left(\ln(x^2+x) - \ln x\right)} = e^{\ln(x^2+x)}e^{(-\ln x)} = e^{\ln(x^2+x)}e^{(\ln x^{(-1)})}$

$= (x^2+x)x^{-1} = \dfrac{(x^2+x)}{x} = x + 1$

Then $f'(x) = \boxed{1}$.

(c) $f(x) = \ln\left(\dfrac{xe^x}{\sqrt{e^{7x}}}\right) = \ln\left(xe^x\right) - \ln\left(\sqrt{e^{7x}}\right) = \ln x + \ln\left(e^x\right) - \ln\left(\left(e^{7x}\right)^{\frac{1}{2}}\right)$

$= \ln x + x - \dfrac{1}{2}\ln\left(e^{7x}\right) = \ln x + x - \dfrac{1}{2}(7x)$

Then $f'(x) = \boxed{\dfrac{1}{x} + 1 - \dfrac{7}{2}}$.

32. Let $f(x) = x^{\cos x}$. Compute $f'(x)$.

Set $y = x^{\cos x}$

Then $\ln y = \ln\left(x^{\cos x}\right) = \cos x \ln x$.

Implicit differentation yields

$\dfrac{1}{y}\dfrac{dy}{dx} = \cos x\dfrac{1}{x} + \ln x(-\sin x)$.

Solve for $\dfrac{dy}{dx} = y\left(\dfrac{\cos x}{x} - \ln x \sin x\right) = \boxed{x^{\cos x}\left(\frac{\cos x}{x} - \ln x \sin x\right)}$

33. Let $f(x) = (\tan x)^x$. Compute $f'(x)$. Set $y = (\tan x)^x$

Then $\ln y = \ln\left((\tan x)^x\right) = x \ln(\tan x)$.

Implicit differentation yields

$$\frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{\tan x}\left(\sec^2 x\right)\right) + \ln(\tan x)(1).$$

Solve for $\dfrac{dy}{dx} = y\left(x\left(\dfrac{\sec^2 x}{\tan x}\right) + \ln(\tan x)\right) = \boxed{(\tan x)^x\left(\frac{x\sec^2 x}{\tan x} + \ln(\tan x)\right)}$

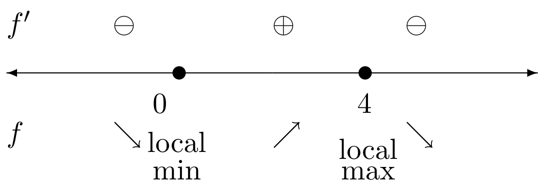
34. Let $f(x) = x^4 e^{-x}$. For this function, discuss domain, vertical and horizontal asymptote(s), interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve.

Take my word that $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = +\infty$

- Domain= $\mathbb{R}$. Note that $f(x) = \dfrac{x^4}{e^x}$ and e^x is never zero in the denominator.
- It has no vertical asymptotes.
- There is a horizontal asymptote for this f at $y = 0$ because $\lim_{x\to\infty} f(x) = 0$.
- First Derivative Information

We compute $f'(x) = x^4 e^{-x}(-1) + e^{-x}(4x^3) = e^{-x}(4x^3 - x^4) = e^{-x}x^3(4 - x)$ and set it equal to 0 and solve for x to find critical numbers. The critical points occur where f' is undefined (never here) or zero. The latter happens when $x = 0$ or $x = 4$.

Using sign testing/analysis for f',



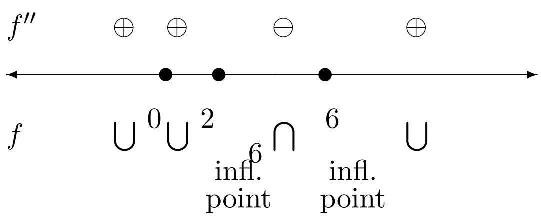
So f is increasing on the interval $(0, 4)$; and f is decreasing on $(-\infty, 0)$ and $(4, \infty)$. Moreover, f has a local max at $x = 4$ with $f(4) = 256e^{-4}$, and a local min at $x = 0$ with $f(0) = 0$.

- Second Derivative Information

Recall, $f'(x) = e^{-x}(4x^3 - x^4)$.

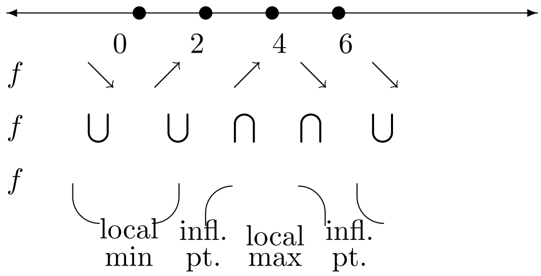
Next $f'' = e^{-x}(12x^2 - 4x^3) - e^{-x}(4x^3 - x^4) = e^{-x}(12x^2 - 4x^3 - 4x^3 + x^4) = e^{-x}x^2(12 - 8x + x^2) = e^{-x}x^2(x-2)(x-6)$. Setting $f'' = 0$ we solve for our possible inflection points $x = 0, x = 2$, or $x = 6$.

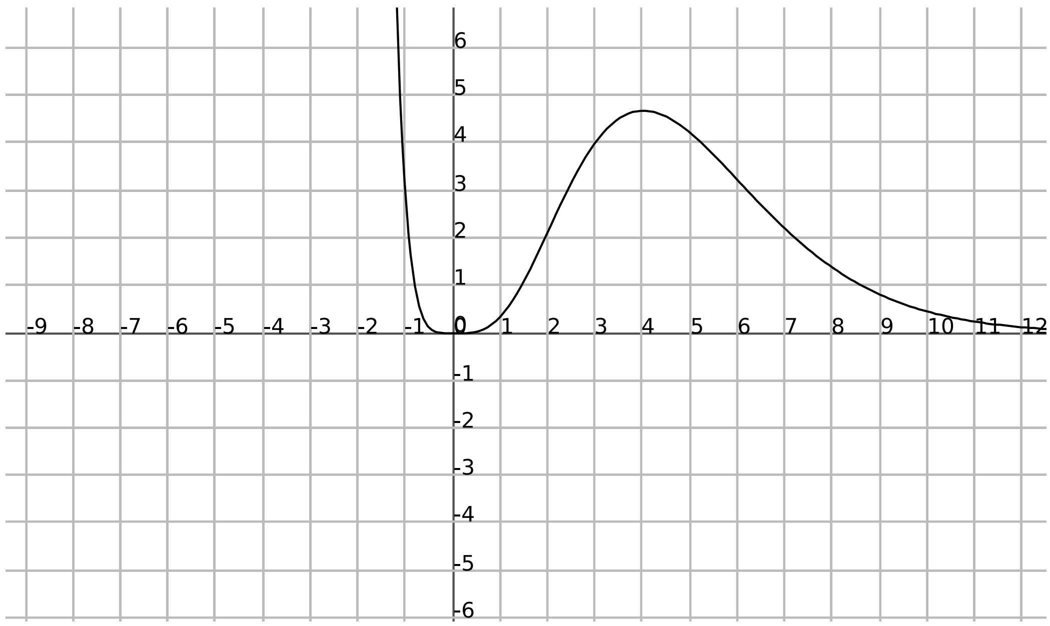
Using sign testing/analysis for f'',



So f is concave down on the interval $(2, 6)$ and concave up on $(-\infty, 2)$ and $(6, \infty)$, with inflection points at $x = 2$ and $x = 6$.

- Piece the first and second derivative information together





Areas between Curves and Volumes of Revolution

SEE ME FOR THE SKECTHES OF THE REGIONS IF YOU HAVE ANY QUESTIONS!

35. Consider the region in the plane bounded by the curves $y = e^{x+1}$, $y = e^{2x}$, and the y-axis.

(a). Find the area of this region.

First, the two curves intersect when $e^{x+1} = e^{2x}$ or when $\ln e^{x+1} = \ln e^{2x}$ so that $x + 1 = 2x \implies x = 1$. The other boundary line is the y-axis, which is $x = 0$. Between $x = 0$ and $x = 1$, the curve $y = e^{x+1}$ lies above the curve $y = e^{2x}$.

Then Area= $\displaystyle\int_0^1 e^{x+1}-e^{2x}\ dx = e^{x+1}-\frac{1}{2}e^{2x}\Big|_0^1 = e^2-\frac{e^2}{2}-\left(e^1-\frac{1}{2}e^0\right) = \boxed{\frac{e^2}{2}-e^1+\frac{1}{2}}$

(b). Rotate this region about the x-axis. What is the volume of the resulting solid?

Volume= $\displaystyle\int_0^1 \pi((\text{outer radius})^2-(\text{inner radius})^2)\ dx = \int_0^1 \pi\left(\left(e^{x+1}\right)^2-\left(e^{2x}\right)^2\right)\ dx$

$\displaystyle = \int_0^1 \pi\left(e^{2x+2}-e^{4x}\right)\ dx = \pi\left(\frac{e^{2x+2}}{2}-\frac{e^{4x}}{4}\right)\Big|_0^1 = \pi\left(\left(\frac{e^4}{2}-\frac{e^4}{4}\right)-\left(\frac{e^2}{2}-\frac{e^0}{4}\right)\right)$

$\displaystyle = \boxed{\pi\left(\frac{e^4}{4}-\frac{e^2}{2}+\frac{1}{4}\right)}$

36. Consider the region enclosed by $y=e^{-x}$, $y=e^x$, and $x=2$ and rotate it about the x–axis. What is the volume of the resulting solid?

Volume= $\displaystyle\int_0^2 \pi((\text{outer radius})^2-(\text{inner radius})^2)\ dx = \int_0^2 \pi\left((e^x)^2-\left(e^{-x}\right)^2\right)\ dx$

$\displaystyle = \int_0^2 \pi\left(e^{2x}-e^{-2x}\right)\ dx = \pi\left(\frac{e^{2x}}{2}-\frac{e^{-2x}}{-2}\right)\Big|_0^2 = \pi\left(\left(\frac{e^4}{2}+\frac{e^{-4}}{2}\right)-\left(\frac{e^0}{2}+\frac{e^0}{2}\right)\right)$

$\displaystyle = \boxed{\pi\left(\frac{e^4}{2}+\frac{1}{2e^4}-1\right)}$

37. Consider the region enclosed by $y=\frac{1}{x}$, $y=0$, $x=1$ and $x=3$ and rotate it about the x–axis. What is the volume of the resulting solid?

Volume= $\displaystyle\int_1^3 \pi(f(x))^2\ dx = \int_1^3 \pi\left(\frac{1}{x}\right)^2\ dx = \int_1^3 \pi x^{-2}\ dx = \pi\frac{x^{-1}}{-1}\Big|_1^3 = -\frac{\pi}{x}\Big|_1^3$

$\displaystyle = -\frac{\pi}{3}-\left(-\frac{\pi}{1}\right) = -\frac{\pi}{3}+\pi = \boxed{\frac{2\pi}{3}}$

38. Find the area enclosed by $y=e^x, y=e^{3x}$ and $x=1$.

The curves $y=e^x$ and $y=e^{3x}$ intersect when $e^x=e^{3x}$ or when $\ln(e^x)=\ln(e^{3x}) \Longrightarrow x = 3x \Longrightarrow x=0$.

Finally, the area $\displaystyle A = \int_0^1 e^{3x}-e^x\ dx = \frac{1}{3}e^{3x}-e^x\Big|_0^1 = \left(\frac{1}{3}e^3-e^1\right)-\left(\frac{1}{3}e^0-e^0\right) =$ $\displaystyle\left(\frac{1}{3}e^3-e^1\right)-\left(\frac{1}{3}-1\right) = \boxed{\frac{1}{3}e^3-e+\frac{2}{3}}$

Properties of e^x and $\ln x$

39. Simplify each of the following

(a) $\ln(e^{\ln e}) = \ln(e^1) = \boxed{1}$ or $\ln(e^{(\ln e)}) = (\ln e)(\ln e) = (1)(1) = \boxed{1}$

(b) $\ln\left|\ln\frac{1}{e}\right| = \ln\left|\ln e^{-1}\right| = \ln|-1| = \ln 1 = \boxed{0}$

40. Solve each of the the following equations for x:

(a) $\ln(\ln x) = 1$
First $e^{\ln(\ln x)} = e^1 \Longrightarrow \ln x = e$. Then, $e^{\ln x} = e^e \Longrightarrow \boxed{x = e^e}$.

(b) $\ln(x^2) = 2 + \ln x$
Note that $x > 0$ is required here.
First $e^{\ln(x^2)} = e^{2+\ln x} = e^2 e^{\ln x} = e^2 x \Longrightarrow x^2 = e^2 x$.
Then, $x^2 - e^2 x = 0 \Longrightarrow x(x - e^2) = 0 \Longrightarrow x = 0$ or $x = e^2$. Since we must have $x > 0$ we can ignore the solution $x = 0$, so $\boxed{x = e^2}$.

(c) $e^{3x-4} = 7$

First $\ln\left(e^{3x-4}\right) = \ln 7 \Longrightarrow 3x - 4 = \ln 7 \Longrightarrow \boxed{x = \dfrac{\ln 7 + 4}{3}}$.

41. Decide whether each statement is True or False. Explain why or why not.

(a) $(e^x)^2 = e^{x^2}$
False, because $(e^x)^2 = e^{2x} \neq e^{x^2}$. "piggy-back exponents multiply"

(b) $\ln 5 - \ln 3 = \ln 2$

False, because $\ln 5 - \ln 3 = \ln\left(\dfrac{5}{3}\right) \neq \ln 2$.

(c) $(\ln x)(\ln x) = \ln(x^2)$
False because$(\ln x)(\ln x) = (\ln x)^2 \neq 2\ln x$ which equals $\ln(x^2)$.