Name:_____

Amherst College

DEPARTMENT OF MATHEMATICS

Math 111

Midterm Exam #3

December 5, 2014

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.

• You need *not* simplify algebraically complicated answers for the derivative section. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, e^0 should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		20
2		25
3		10
4		15
5		20
6		10
Total		100

1. [20 Points] Compute each of the following derivatives.

(a) $f'(x)$ where $f(x) = \displaystyle\int_{\cos x}^{7} \frac{1}{e^t + 5t}\, dt$

(b) y' where $y = e^9 + \dfrac{e^{9x}}{9} + e^{\frac{9}{x}} - \dfrac{1}{9e^x} + \dfrac{9}{e^x} + \dfrac{e^x}{e^{9x}} + e^{9-x} + \dfrac{1}{9 - e^x} + e^x \cdot e^{9x} + e^{\sqrt{9-x}}$.

Do **not** simplify here.

1. (Continued) Compute the following derivative.

(c) $\dfrac{dy}{dx}$ where $e^{x^2y} + e = y^2 + 1 + \tan x$. Simplify where possible.

2. [25 Points] Compute each of the following integrals. Simplify your answers.

(a) $\displaystyle\int \left(e^{3x} + \frac{1}{e^{2x}}\right)\left(e^{x} + \frac{1}{e^{5x}}\right)\, dx$

(b) $\displaystyle\int \frac{1}{e^{4x}\left(1 + e^{-4x}\right)^{4}}\, dx$

(c) $\displaystyle\int_{\pi}^{3\pi} \sin\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)\, dx$

2. [Continued] Compute each of the following integrals. Simplify your answers.

(d) $\displaystyle\int_{-2}^{-1} x\,(x+1)^7\,dx$

(e) $\displaystyle\int \frac{x^{\frac{3}{5}} - x^{\frac{1}{4}}}{\sqrt{x}}\,dx$

3. [10 Points] Find the function $f(x)$ that satisfies $f'(x) = \dfrac{\sec^2 x}{\sqrt{3+\tan x}}$ and $f\left(\dfrac{\pi}{4}\right) = 9$.

4. [15 Points] You need to construct a box with a square base with a fixed volume of 24 cubic feet. The material for the bottom and top costs \$3 per square foot, and the material for the sides costs \$1 per square foot. What are the **dimensions** that minimize the cost required to build such a box? What is that **minimum cost**?

(Don't forget to state the common sense bounds, that is, the domain of the function that you are maximizing or minimizing.)

5. [20 Points] Compute $\int_1^3 5 - x^2 \, dx$ using each of the following **two** different methods:

(a) Fundamental Theorem of Calculus.

(b) Limit definition of the definite integral ***.

*** Recall $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, and $\sum_{i=1}^{n} 1 = n$

6. [10 Points] A moving object has velocity $v(t) = t^2 - 1$ feet per second, at time t seconds. Compute the **Total Distance** travelled by this object from time $t = 0$ to $t = 3$ seconds.

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute the following limit:

$$\lim_{n\to\infty} \frac{5}{n}\left(\sqrt{4+\frac{5}{n}}+\sqrt{4+\frac{10}{n}}+\sqrt{4+\frac{15}{n}}+\sqrt{4+\frac{20}{n}}+\ldots+\sqrt{4+5}\right)$$