Math 111 Midterm Exam #3 November 30, 2012

1. [20 Points] **Differentiate** each of the following functions. You **do not** need to simplify your answers.

(a) $f(x) = \displaystyle\int_{\sec x}^{7} \sqrt{\cos t + 7e^t}\ dt$

(b) $f(x) = \tan\left(e^x + \sqrt{x}\right) + e^{\tan\sqrt{x}} + \sqrt{e^x + \tan x}$

(c) $f(x) = e^x + x^e + ex + e^e + e^{(e^x)} + (x^e)^e + e^{\frac{1}{x}} - \dfrac{1}{e^x}.$

2. [25 Points] Compute each of the following integrals. Simplify your answers.

(a) $\displaystyle\int \left(e^{7x} + \frac{1}{e^{4x}}\right)^2 dx$

(b) $\displaystyle\int_0^1 \frac{e^x}{\sqrt{e^x + 8}}\ dx$

(c) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos x}{\sin^3 x}\ dx$

(d) $\displaystyle\int x\ (x-1)^{\frac{5}{7}}\ dx$

3. [10 Points] Find the function $f(x)$ that satisfies $f'(x) = \dfrac{e^{\sqrt{\tan x}}\ \sec^2 x}{\sqrt{\tan x}}$ and $f\left(\dfrac{\pi}{4}\right) = 1.$

4. [15 Points] You need to construct a box with a square base with a fixed volume of 24 cubic feet. The material for the bottom and top costs \$3 per square foot, and the material for the sides costs \$1 per square foot. What are the **dimensions** that minimize the cost required to build such a box? What is that **minimum cost**?

(Don't forget to state the common sense bounds, that is, the domain of the function that you are maximizing or minimizing.)

5. [20 Points] Compute $\int_1^3 x^2 - 3x \; dx$ using each of the following **two** different methods:

(a) Fundamental Theorem of Calculus.

(b) Riemann Sums and the limit definition of the definite integral ***.

*** Recall $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, and $\sum_{i=1}^{n} 1 = n$

6. [10 Points] A moving object has velocity $v(t) = 2t - 6$ feet per second, at time t seconds. Compute the **Total Distance** travelled by this object from time $t = 0$ to $t = 4$ seconds.

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute $\lim_{n\to\infty} \frac{e^{\left(1+\frac{1}{n}\right)} + e^{\left(1+\frac{2}{n}\right)} + e^{\left(1+\frac{3}{n}\right)} + \ldots + e^2}{n}$

OPTIONAL BONUS #2 Compute $\int \sin^3 x \; dx$