Name:_____

Amherst College

DEPARTMENT OF MATHEMATICS

Math 111

Midterm Exam #2

October 24, 2014

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted. Do not access any webpages during this exam.

• You need *not* simplify algebraically complicated answers for the derivative section. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, and $4^{\frac{3}{2}}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

• If you actually read these directions, draw a ghost at the bottom of the page.

Problem	Score	Possible Points
1		10
2		25
3		10
4		20
5		10
6		15
7		10
Total		100

1. [10 Points] Compute each of the following **limits**. Justify your answers. Show your work.

(a) $\lim_{x\to\infty} \dfrac{x^9 + 8x^7 + 6x^5 + 4}{3x^2 + 1}$

(b) $\lim_{x\to\infty} \dfrac{x^2 - x + 1}{2x^5 + 7x^2 + 3}$

2. [25 Points] **Differentiate** each of the following functions. You **do not** need to simplify your answers. Please do not waste time simplifying your derivative.

(a) $f(x) = \frac{5}{6}x + x^{\frac{5}{6}} + \frac{1}{x^{\frac{5}{6}}} + \frac{6}{5} + \frac{5}{6x^6} - \frac{6}{5x}$

(b) $f(x) = \left(\frac{\frac{3}{x^2} + x^3}{x^{\frac{2}{3}} + \frac{3}{2}x} \right)^{\frac{2}{3}}$

(c) $f(x) = \frac{\sqrt{x} + \sec\sqrt{x}}{\sqrt{1 + \sec x}}$

2. (Continued) **Differentiate** each of the following functions. You **do not** need to simplify your answers. Please do not waste time simplifying your derivative.

(d) $f(x) = \cos\left(\tan^2\left(\frac{3}{x^5}\right)\right) + \sin^2\left(\cos\left(\frac{x^3}{5}\right)\right)$

(e) $f(x) = \left(\frac{1}{x^3} + \pi\right)^{\frac{5}{7}} \cdot \left(x^4 - \frac{1}{x^7}\right)^{-5}$

3. [10 Points] Find the **absolute maximum** and **absolute minimum value(s)** of the function

$$F(x) = x\sqrt{4 - x^2} \qquad \text{on the interval} \qquad [-1, 2].$$

4. [20 Points] Let $f(x) = \dfrac{-x^2+3x}{(x-2)^2} = \dfrac{-x^2+3x}{x^2-4x+4}$.

For this function, discuss domain, vertical and horizontal asymptote(s), interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve. **Hint:**

Take my word for it that (you do **not** have to compute these)

$$\boldsymbol{f'(x) = \dfrac{x-6}{(x-2)^3}} \quad \text{and} \quad \boldsymbol{f''(x) = \dfrac{-2x+16}{(x-2)^4}}.$$

5. [10 Points] Consider the equation $\cos(xy^2) + 2 = y^3 + \sin x$.

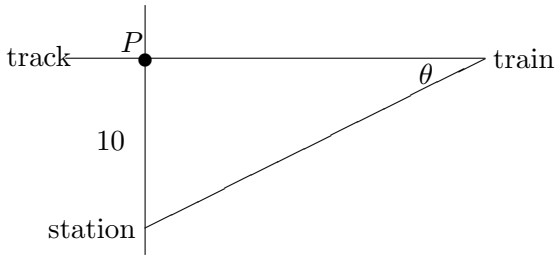
(a) Compute $\dfrac{dy}{dx}$.

(b) Compute the equation of the tangent line to this curve at the point $(\pi, 1)$.

6. [15 Points] Consider a point P on a train track. Suppose a train depot station is 10 feet directly south from this point P. The train is travelling east at 6 feet per second. Consider the angle as shown in the diagram. How fast is this angle changing when 2 seconds has passed since the train passed point P.

• Diagram

The picture at arbitrary time t is:



7. [10 Points]

(a) Let $f(x) = \sin^3(4x) + \sec(4x) - 8\sin(2x)$. Compute $f'\left(\frac{\pi}{12}\right)$. Simplify.

(b) Let $f(x) = \cos(2x) + \frac{1}{\tan^2 x} + \sin\left(x - \frac{\pi}{4}\right)$. Compute $f'\left(\frac{\pi}{4}\right)$. Simplify.