

1. [12 Points] Compute each of the following **limits**. Justify your answers. Show your work.

(a) $\lim_{x \rightarrow 0} \frac{8x^2 - x^3}{\sin^2(3x)}$

(b) $\lim_{x \rightarrow \infty} \frac{3x^7 + x^6 - 2012}{x^2 - x + 5}$

2. [18 Points] **Differentiate** each of the following functions. You **do not** need to simplify your answers. Please do not waste time simplifying your derivative.

(a) $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + \frac{1}{1 + \sqrt{x}} + \frac{1}{\sqrt{1 + x}}$

(b) $f(x) = \left(\frac{\cos(7x)}{x^2 + 7 \sin x} \right)^{\frac{7}{8}}$

(c) $f(x) = \frac{1}{\left(\frac{1}{x^5} + \sqrt{x^2 - 4} \right)^{\frac{3}{7}}}$

3. [10 Points] Find the **absolute maximum** and **absolute minimum value(s)** of the function

$$F(x) = (x - 1)^2(2x - 10)^2 \quad \text{on the interval} \quad [0, 4].$$

4. [25 Points] Let $f(x) = \frac{x^2 - 9}{x^2 - 4}$.

For this function, discuss domain, vertical and horizontal asymptote(s), interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve. **Hint:**

Take my word for it that (you do **not** have to compute these)

$$f'(x) = \frac{10x}{(x^2 - 4)^2} \quad \text{and} \quad f''(x) = \frac{-10(3x^2 + 4)}{(x^2 - 4)^3}.$$

5. [10 Points] Consider the equation $\sin(x^2y) + 6 \tan x + 1 = y^3$.

(a) Compute $\frac{dy}{dx}$.

(b) Compute the equation of the tangent line to this curve at the point $(0, 1)$.

6. [15 Points] The top of a ten foot ladder is sliding down a vertical wall at the rate of one foot every second. Consider the angle formed by the bottom of the ladder and the ground. How fast is this angle changing when the top of the ladder is three feet above the ground?

7. [10 Points] Let $f(x) = \frac{1}{2 \tan^2 x} + \cos^2 x + \sec(2x)$. Find $f'(\frac{\pi}{6})$.

Simplify your answer to a single real number.

OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

OPTIONAL BONUS Two carts, A and B , are connected by a rope 39 feet long that passes over a pulley P on the ceiling. The point Q is on the floor 12 feet directly beneath P and between the carts. Cart A is being pulled away from Q at a speed of 2 feet per second. How fast is cart B moving toward Q at the instant when cart A is 5 feet from Q ?