Math 111 **Midterm Exam #1** **February 17, 2012**

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [30 Points] Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(**a**) $\displaystyle\lim_{x\to-3} \frac{x^2-2x-15}{x^2+x-6} =$ (**b**) $\displaystyle\lim_{x\to5} \frac{x^2-2x-15}{|5-x|} =$

(**c**) $\displaystyle\lim_{x\to-3} \frac{x^2+2x-3}{x^2-4x-12} =$ (**d**) $\displaystyle\lim_{x\to-6} \frac{\frac{x}{x+2}-\frac{x-3}{x}}{x+6} =$

(**e**) $\displaystyle\lim_{x\to2} \frac{x+7}{x-2} =$ (**f**) $\displaystyle\lim_{x\to3} \frac{g(x^2)-7}{(g(x))^2-1} =$ where $g(x)=x-2$

2. [13 Points] Prove that $\displaystyle\lim_{x\to3} 1-5x=-14$ using the $\varepsilon-\delta$ definition of the limit.

3. [15 Points] Suppose that $f(x)=\sqrt{3-x+x^2}$. Compute $f'(x)$ using the **limit definition of the derivative**.

4. [10 Points] Suppose that $f(x)=x^3+7x^2-4x+9$. Write the **equation of the tangent line** to the curve $y=f(x)$ when $x=-1$. **Use the limit definition of the derivative when computing the derivative.**

5. [12 Points] Suppose that f and g are functions, **and**

- $\displaystyle\lim_{x\to7} f(x)=5$
- $\displaystyle\lim_{x\to7} g(x)=-3$
- $f(5)=7$
- $g(x)$ is continuous at $x=7$.
- $f(x)$ is **NOT** continuous at $x=7$.

Evaluate the following quantities and fully **justify** your answers. Do **not** just put down numbers.

(**a**) $\displaystyle\lim_{x\to7} \sqrt{3f(x)-7g(x)} =$ (**b**) $g(7)=$

(**c**) $g\circ f(5)=$ (**d**) Does $f(7)=5$? Justify your answer.

6. [20 Points] Consider the function defined by

$$f(x) = \begin{cases} 3 & \text{if } x > 12 \\ \sqrt{x-3} & \text{if } 3 < x \le 12 \\ 1 & \text{if } x = 3 \\ 6 - 2x & \text{if } 0 < x < 3 \\ 16 - x^2 & \text{if } -4 < x \le 0 \\ \dfrac{1}{x+4} & \text{if } x < -4 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$.

(b) State the **Domain** of the function $f(x)$.

(c) Compute $\begin{cases} \lim_{x\to 0^+} f(x) = \\ \lim_{x\to 0^-} f(x) = \\ \lim_{x\to 0} f(x) = \end{cases}$

(d) Compute $\begin{cases} \lim_{x\to 3^+} f(x) = \\ \lim_{x\to 3^-} f(x) = \\ \lim_{x\to 3} f(x) = \end{cases}$

(e) Compute $\begin{cases} \lim_{x\to -4^+} f(x) = \\ \lim_{x\to -4^-} f(x) = \\ \lim_{x\to -4} f(x) = \end{cases}$

(f) State the value(s) at which f is discontinuous. Justify your answer(s) using the definition of continuity.

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Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Let $f(x) = \dfrac{1}{\sqrt{x^3 - 4x^2 + x - 7}}$. Compute $f'(x)$ using the limit definition of the derivative.

OPTIONAL BONUS #2 Compute $\displaystyle\lim_{x\to 1} \frac{\dfrac{1}{\sqrt{2-x}} - \dfrac{2}{\sqrt{3+x}}}{\dfrac{7}{\sqrt{50-x}} - \dfrac{6}{\sqrt{x+35}}}$