Math 111 Review Packet for Exam #1

Limit Practice Problems

Evaluate the following limits. Be clear if the limit equals a finite value, Does Not Exist, or is $+\infty$ or $-\infty$. Always justify your work:

1. $\lim_{w \to 0} \frac{16}{w} =$

2. $\lim_{t \to 2} \frac{3-t}{t-2} =$

3. $\lim_{t \to 2} \frac{3-t}{(t-2)^2} =$

4. $\lim_{x \to 4} \frac{(x+2)^2}{x^2-3x-4} =$

5. $\lim_{x \to 4} \frac{x-4}{x^2-3x-4} =$

6. $\lim_{x \to 4} \frac{x^2-2x-8}{x^2-3x-4} =$

7. $\lim_{x \to 1} \frac{x^2-4x-12}{x^2-3x-18} =$

8. $\lim_{x \to 0} \frac{x^2-4x-12}{x^2-3x-18} =$

9. $\lim_{x \to -3} \frac{x+2}{x+3} =$

10. $\lim_{x \to -2} \frac{x^2-4x-12}{x^2-3x-18} =$

11. $\lim_{x \to 0} \frac{x^2-4x}{x^2-7x} =$

12. $\lim_{x \to 3} \frac{x^2-9}{|x-3|} =$

13. $\lim_{x \to -5} \frac{x^2+6x+5}{|x+5|} =$

14. $\lim_{t \to 1} \frac{t^2-1}{t^2-11t+10} =$

15. $\lim_{t \to 1} \frac{t^2}{t^2+t-1} =$

16. $\lim_{t \to -1} \frac{2009(t^2+6t+5)}{t^2+t} =$

17. $\lim_{x \to 9} \frac{x^2 - 10x + 9}{x^2 + x - 90} =$

18. $\lim_{t \to 1} t^{500} + t^{400} + t^{300} + t^{200} + t^{100} =$

19. $\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 2x - 3} =$

20. $\lim_{x \to 1} \frac{x - 1}{\sqrt{x + 3} - 2} =$

21. $\lim_{x \to 9} \frac{9x - x^2}{3 - \sqrt{x}} =$

22. $\lim_{x \to -1} \frac{5}{1 - x} =$

23. $\lim_{x \to 5} \frac{6x}{5 - x} =$

24. $\lim_{x \to 2} \frac{x^2 - 9x + 14}{x^2 - 4x + 4} =$

25. $\lim_{x \to 2} \frac{x^2 - 4}{|x - 2|} =$

26. $\lim_{x \to 3} \frac{\sqrt{x + 6} - 3}{x^2 - x - 6} =$

27. $\lim_{x \to 7} \frac{\frac{1}{7} - \frac{1}{x}}{x - 7} =$

28. $\lim_{x \to -6} \frac{\frac{1}{2 - x} - \frac{1}{8}}{x + 6} =$

29. $\lim_{x \to 3} \frac{\sqrt{x + 1} - 2}{3 - x} =$

30. $\lim_{x \to 7} \frac{x^2 - 49}{2 - \sqrt{x - 3}} =$

31. $\lim_{x \to 5} \frac{\frac{1}{\sqrt{x + 20}} - \frac{1}{5}}{x - 5} =$ Challenge!

Functions and Limit Practice Problems Evaluate the following limits:

32. Let $g(x) = 2x + 1$. Compute $\displaystyle\lim_{x\to 1} \frac{x-1}{g(x^2)-3} =$

33. Let $G(u) = u^2 + u$. Compute $\displaystyle\lim_{u\to 2} \frac{u^2-2u}{G(u-3)} =$

34. Let $h(y) = y^2 - 3$. Compute $\displaystyle\lim_{x\to -2} \frac{x+2}{h(2x)-h(x+6)} =$

35. Let $f(t) = \dfrac{1}{t}$. Compute $\displaystyle\lim_{t\to 4} \frac{f(t-3)-4f(t)}{t-4} =$

36. Compute $\displaystyle\lim_{x\to -6} \frac{f(x^2)+5x-8}{[f(x)]^2+5x+14} =$ where $f(x) = x+2$

More Functions

37. Let $f(x) = \sqrt{x}$, $g(x) = x^2 + 4$, and $h(x) = \dfrac{1}{x}$. Compute (and simplify, if possible) the following:

    (a) $f \circ g(x) =$
    (b) $g \circ f(x) =$
    (c) $h \circ g \circ f(x) =$
    (d) $g \circ g(x) =$

$\varepsilon - \delta$ **Definition of the Limit**

Use the $\varepsilon - \delta$ definition for limits to prove each of the following:

38. $\displaystyle\lim_{x\to 2} 7x - 6 = 8.$

39. $\displaystyle\lim_{x\to -7} 2 - \frac{3}{7}x = 5.$

40. $\displaystyle\lim_{x\to -2} 2x + 1 = -3.$

41. $\displaystyle\lim_{x\to 3} 1 - 4x = -11.$

42. $\displaystyle\lim_{x\to -3} 1 - 5x = 16.$

43. $\displaystyle\lim_{x\to -14} \frac{4}{7}x + 3 = -5.$

Derivatives Use the limit definition of the derivative to calculate the derivative for each of the following functions:

44. $f(x) = 3 - 9x^2$
45. $f(x) = x^3$
46. $f(x) = \dfrac{1}{x^2}$
47. $f(x) = \sqrt{x - 7}$
48. $f(x) = \dfrac{1}{\sqrt{x}}$
49. $f(x) = \dfrac{3 - x}{x - 4}$
50. $f(x) = \dfrac{3x - 1}{2 - 5x}$

Tangent Lines Please use the limit definition for the derivative when comp uting the derivatives in this section.

51. Find an equation for the tangent line to the graph of $f(x) = x - 2x^2$ at the point $(1, -1)$
52. Find an equation for the tangent line to the graph of $f(x) = \sqrt{x}$ at $x = 4$
53. At which point(s) does the graph of $f(x) = -x^2 + 13$ have a horizontal tangent line?
54. At which point(s) of the graph of $f(x) = -x^3 + 13$ is the slope of the tangent line equal to -27? What's the picture representing this problem?
55. There are two points on the graph of the curve $y = -x^2 + 7$ whose tangent line to the graph at those points passes through the point $(0, 11)$. Find those two points.
56. Find the equation of the line passing through $(2, 3)$ which is perpendicular to the tangent to the curve $y = x^3 - 3x + 1$ at the point $(2, 3)$.
57. Find the equation of the tangent line to the curve $y = x^3 + x$ at the point(s) where the slope equals 4.
58. Find an equation for the tangent line to the graph of $f(x) = \dfrac{1}{x - 1}$ at the point $(0, -1)$.

Piece-wise defined functions

Consider each of the following piecewise defined functions. Answer the related questions. *Justify* your answers please.

59. Let $f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 \leq x \leq 2 \\ 8 - x & \text{if } x > 2 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Justify your answer(s) using the definition of continuity.

Evaluate:

$\lim_{x \to 2} f(x) =$

$\lim_{x \to 0} f(x) =$

60. Let $f(x) = \begin{cases} x - 1 & \text{if } x < 2 \\ 1 & \text{if } 2 < x < 4 \\ 3 & \text{if } x = 4 \\ \sqrt{x} & \text{if } x > 4 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Justify your answer(s) using the definition of continuity.

Evaluate:

$\lim_{x \to 0} f(x) =$

$\lim_{x \to 2} f(x) =$

$\lim_{x \to 4} f(x) =$

$f(4) =$

61. Let $h(x) = \begin{cases} \dfrac{8}{x+2} & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ \dfrac{1}{2}x - 4 & \text{if } 0 < x \leq 16 \\ \sqrt{x} & \text{if } x > 16 \end{cases}$

Sketch the graph. Find the numbers at which h is discontinuous. Justify your answer(s) using the definition of continuity.

Evaluate:

$\lim_{x \to -2} h(x) =$

$\lim_{x \to 0} h(x) =$

$\lim_{x \to 16} h(x) =$

62. Let $F(x) = \begin{cases} \dfrac{1}{x-4} & \text{if } x > 4 \\ 3 - x & \text{if } 0 < x < 4 \\ 3 - x^2 & \text{if } x < 0 \end{cases}$

Sketch the graph. Find the numbers at which F is discontinuous. Justify your answer(s) using the definition of continuity.

Evaluate:

$\lim_{x \to 0} F(x) =$

$\lim_{x \to 4} F(x) =$

63. Let $f(x) = \begin{cases} 2 & \text{if } x \geq 11 \\ \sqrt{x-7} & \text{if } 7 < x < 11 \\ 1 & \text{if } x = 7 \\ 7 - x & \text{if } 0 < x < 7 \\ 16 - x^2 & \text{if } -4 < x \leq 0 \\ \dfrac{1}{x+4} & \text{if } x < -4 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Justify your answer(s) using the definition of continuity.

Evaluate:

$\lim_{x \to -4} f(x) =$

$\lim_{x \to 0} f(x) =$

$\lim_{x \to 7} f(x) =$

$\lim_{x \to 11} f(x) =$