

Name: \_\_\_\_\_

**Amherst College**  
**DEPARTMENT OF MATHEMATICS**  
**Math 111**  
**Section 01**  
**Midterm Exam #1**  
**September 26, 2014**

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		30
2		15
3		15
4		10
5		10
6		20
Total		100

**1.** [30 Points] Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value,  $+\infty$  or  $-\infty$ , or Does Not Exist.

(a)  $\lim_{x \rightarrow -6} \frac{x^2 + 4x - 21}{x^2 - 5x - 6} =$

(b)  $\lim_{x \rightarrow 6} \frac{x^2 - 13x + 42}{|6 - x|} =$

(c)  $\lim_{x \rightarrow -3} \frac{f(2x) - 3f(x) - 7}{f(x + 2) - 2} =$  where  $f(x) = x^2 + 1$

**1.** (Continued) Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value,  $+\infty$  or  $-\infty$ , or Does Not Exist.

(d)  $\lim_{x \rightarrow -5} \frac{x^2 - x - 30}{x^2 + 10x + 25} =$

(e)  $\lim_{x \rightarrow 4} \frac{\frac{3-x}{x-5} - \frac{3}{7-x}}{x^2 - x - 12} =$

**2.** [15 Points] Prove that  $\lim_{x \rightarrow 2} 5 - 3x = -1$  using the  $\varepsilon - \delta$  **definition of the limit**.

**3.** [15 Points] Suppose that  $f(x) = \frac{5 - 2x}{1 + 6x}$ . Compute  $f'(x)$  using the **limit definition of the derivative**.

**4.** [10 Points] Suppose that  $f(x) = \sqrt{6-x}$ . Write the **equation of the tangent line** to the curve  $y = f(x)$  where  $x = -3$ .

\*\*Use the limit definition of the derivative when computing the derivative.\*\*

5. [10 Points] Suppose that  $G$  and  $H$  are functions, **and**

- $\lim_{x \rightarrow 5} G(x) = 6$
- $\lim_{x \rightarrow -9} H(x) = -4$
- $\lim_{x \rightarrow 8} G(x) = 7$
- $G(x)$  is continuous at  $x = 8$ .
- $H(x)$  is continuous at  $x = 7$ .
- $G(5) = -9$
- $H(7) = -9$

Answer the following questions or evaluate the following quantities and fully **justify** your answers.

(a) Compute  $G(8) =$

(b) Compute  $\lim_{x \rightarrow 7} H(x) =$

(c) Compute  $H \circ G(8) =$

(d) Does  $H(-9) = -4$ ? Yes, No, or Not Enough Information? Why or why not?

(e) Is  $G(x)$  continuous at  $x = 5$ ?

6. [20 Points] Consider the function defined by

$$f(x) = \begin{cases} \sqrt{x-3} & \text{if } x > 3 \\ 5 & \text{if } x = 3 \\ 1 - (x-2)^2 & \text{if } 2 < x < 3 \\ 5 - 2x & \text{if } 0 \leq x < 2 \\ \frac{1}{x+3} & \text{if } x < 0 \end{cases}$$

(a) Carefully sketch the graph of  $f(x)$ .

(b) State the **Domain** of the function  $f(x)$ .



**6.** (Continued) Continue to consider the function defined by

$$f(x) = \begin{cases} \sqrt{x-3} & \text{if } x > 3 \\ 5 & \text{if } x = 3 \\ 1 - (x-2)^2 & \text{if } 2 < x < 3 \\ 5 - 2x & \text{if } 0 \leq x < 2 \\ \frac{1}{x+3} & \text{if } x < 0 \end{cases}$$

(c) Compute  $\lim_{x \rightarrow -3} f(x) =$

(d) Compute  $\lim_{x \rightarrow 0} f(x) =$

(e) Compute  $\lim_{x \rightarrow 2} f(x) =$

(f) Compute  $\lim_{x \rightarrow 3} f(x) =$

(g) State the value(s) at which  $f$  is discontinuous. Justify your answer(s) using definitions or theorems discussed in class.

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# OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1    Compute  $\lim_{x \rightarrow 1} \frac{2|x - 1| - |x + 2| + |x| + |x + 1|}{|x - 1| + |3 - x| - |x + 1|} =$