Math 111 **Midterm Exam** #1 **September 28, 2012** Compact Version

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [30 Points] Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\lim_{x\to -7} \dfrac{x^2+5x-14}{x^2-4x+4} =$ **(b)** $\lim_{x\to 4} \dfrac{x^2-9x+20}{|4-x|} =$

(c) $\lim_{x\to -6} \dfrac{f(x^2)+5x-8}{[f(x)]^2+5x+14} =$ where $f(x)=x+2$ **(d)** $\lim_{x\to 2} \dfrac{x^2+5x-14}{x^2-4x+4} =$

(e) $\lim_{x\to 8} \dfrac{3-\sqrt{x+1}}{x^2-7x-8} =$

2. [13 Points] Prove that $\lim_{x\to 5} 7-2x=-3$ using the $\varepsilon-\delta$ definition of the limit.

3. [15 Points] Suppose that $f(x)=\dfrac{x+7}{x-3}$. Compute $f'(x)$ using the **limit definition of the derivative**.

4. [10 Points] Suppose that $f(x)=5-7x+4x^2-x^3$. Write the **equation of the tangent line** to the curve $y=f(x)$ when $x=1$. **Use the limit definition of the derivative when computing the derivative.**

5. [6 Points] Suppose that f and g are functions, **and**

- $\lim_{x\to 3} f(x)=9$
- $\lim_{x\to 7} g(x)=-6$
- $\lim_{x\to 4} f(x)=7$
- $g(x)$ is continuous at $x=7$.
- $f(x)$ is continuous at $x=4$.

(a) Compute $g\circ f(4) =$ (Do **not** just put down a value. Justify your answer.)

(b) Does $f(3)=9$? Why or why not?

6. [6 Points] Suppose that $f(x)=\sqrt{x+4}$ and $g(x)=x+2$.

(a) Compute **and** graph $f\circ g(x)$. **(b)** Compute **and** graph $g\circ f(x)$.

7. [20 Points] Consider the function defined by

$$f(x) = \begin{cases} \sqrt{x-7} & \text{if } x > 7 \\ 1 & \text{if } x = 7 \\ 7 - x & \text{if } 0 < x < 7 \\ 16 - x^2 & \text{if } -4 < x \leq 0 \\ \dfrac{1}{x+4} & \text{if } x < -4 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$.

(b) State the **Domain** of the function $f(x)$.

(c) Compute $\begin{cases} \lim\limits_{x\to 0^+} f(x) = \\ \lim\limits_{x\to 0^-} f(x) = \\ \lim\limits_{x\to 0} f(x) = \end{cases}$ **(d)** Compute $\begin{cases} \lim\limits_{x\to 7^+} f(x) = \\ \lim\limits_{x\to 7^-} f(x) = \\ \lim\limits_{x\to 7} f(x) = \end{cases}$ **(e)** Compute $\begin{cases} \lim\limits_{x\to -4^+} f(x) = \\ \lim\limits_{x\to -4^-} f(x) = \\ \lim\limits_{x\to -4} f(x) = \end{cases}$

(f) State the value(s) at which f is discontinuous. Justify your answer(s) using definitions or theorems discussed in class.

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1 Compute $\lim\limits_{x\to 2} \dfrac{(4-\sqrt{x+14})(\sqrt{13-x^2}-3)}{(6-\sqrt{40-2x})(\sqrt{x^2+21}-5)} =$

OPTIONAL BONUS #2 Let $f(x) = \sqrt{\dfrac{x^2+1}{7-x^3}}$. Compute $f'(x)$.

OPTIONAL BONUS #3 Compute $\lim\limits_{x\to 0} \dfrac{|x-1|-|x+1|-|x|}{|x|+|2-x|-|x+2|} =$