Extra Example for Cruve Sketching with Vertical and Horizontal Asymptotes

Professor D. Benedetto,

1. $f(x) = \dfrac{x}{x-2}$

• Domain: $f(x)$ has domain $\{x | x \neq 2\}$

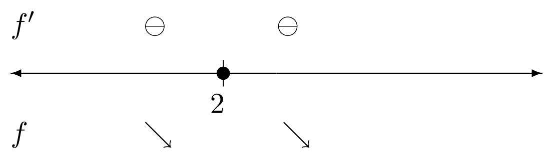
• VA: Vertical asymptote at $x = 2$.

• HA: Horizontal asymptote at $y = 1$ for this $f$ since $\lim_{x \to \pm\infty} f(x) = 1$ because

$$\lim_{x \to \pm\infty} \frac{x}{x-2} \cdot \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)} = \lim_{x \to \pm\infty} \frac{1}{1 - \frac{2}{x}} = 1.$$

• First Derivative Information:

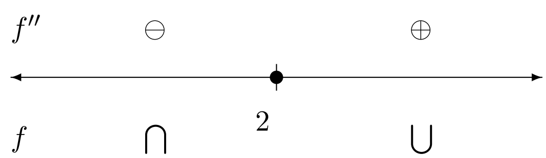
We compute $f'(x) = \dfrac{-2}{(x-2)^2}$ to find critical numbers. The critical points occur where $f'$ is undefined or zero (never here). The former happens when $x = 2$, which was not in the domain of the original function. As a result, there are no critical numbers. Using sign testing/analysis for $f'$ around $x = 2$,




So $f$ is decreasing on $(-\infty, 2)$ and $(2, \infty)$ . Moreover, $f$ has no extreme values.

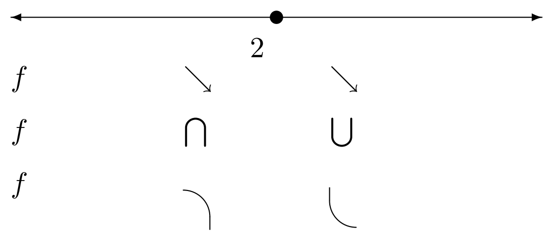
• Second Derivative Information:

Meanwhile, $f'' = \dfrac{4}{(x-2)^3}$. Using sign testing/analysis for $f''$ around $x = 2$,




So $f$ is concave down on $(-\infty, 2)$ and concave up on $(2, \infty)$.

- Piece the first and second derivative information together:




- Sketch: