Math 111 Final Examination December 19, 2012

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.

• You need *not* simplify algebraically complicated answers. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, $e^{\ln 4}$, $\ln(e^7)$, $e^{-\ln 5}$, or $e^{3\ln 3}$ should be simplified.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [20 Points] Evaluate each of the following limits. Please justify your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\displaystyle\lim_{x\to -2} \frac{x^2+3x+2}{x^2+x-2}$ (b) $\displaystyle\lim_{x\to 1^-} \frac{g(x+1)-2x-5}{(x-1)^2}$, where $g(x)=x^2+3$.

(c) $\displaystyle\lim_{x\to\infty} \frac{3-2x^2}{3x^2+5x}$. (d) $\displaystyle\lim_{x\to 5} \frac{25-x^2}{\sqrt{x+4}-3}$ (e) $\displaystyle\lim_{x\to 7} \frac{|7-x|}{x^2-x-42}$

2. [30 Points] Compute each of the following derivatives.

(a) $f'\left(\frac{\pi}{6}\right)$, where $f(x)=\cos^2 x+\tan(2x)+\sin x$. Simplify.

(b) $\displaystyle\frac{d}{dx}\ln\left(\frac{(x^2+1)^{\frac{4}{7}}\,e^{\tan x}}{\sqrt{1+\sqrt{x}}}\right)$ Hint: you might want to simplify before differentiating.

(c) $g'(x)$, where $g(x)=\sqrt{\cos(x^2+e^x)}+\cos\sqrt{x^2+e^x}+e^{\sqrt{x^2+\cos x}}$. Do not simplify here.

(d) $\displaystyle\frac{dy}{dx}$, if $\sin(xy)=\sec x+\cos(e^9)-y$.

(e) $g''(x)$, where $\displaystyle g(x)=\int_x^{2012}\sqrt{\ln t}+\ln\sqrt{t}\;dt$.

(f) $f''(x)$, where $f(x)=\dfrac{x^4}{e^x}$. Simplify here.

3. [25 Points] Compute each of the following integrals.

(a) $\displaystyle\int_{\frac{\pi}{18}}^{\frac{\pi}{9}}\tan(3x)\,dx$ (b) $\displaystyle\int\frac{(3-\sqrt{x})(1+2\sqrt{x})}{x^2}\,dx$ (c) $\displaystyle\int x\sqrt{x+1}\,dx$

(d) $\displaystyle\int_e^{e^3}\frac{4}{x(\ln x)^2}\,dx$ (e) $\displaystyle\int_{\ln 3}^{\ln 8}\frac{e^x}{\sqrt{1+e^x}}\,dx$

4. [10 Points] Give an ε–δ proof that $\displaystyle\lim_{x\to 3} 7-5x=-8$.

5. [10 Points] Let $f(x)=\dfrac{1}{x^2}$. Calculate $f'(x)$, using the **limit definition** of the derivative.

6. [15 Points] Compute $\displaystyle\int_1^3 4 - x^2\, dx$ using each of the following **two** different methods:

(a) Fundamental Theorem of Calculus,

(b) Riemann Sums and the limit definition of the definite integral.

***Recall $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ and $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ and $\displaystyle\sum_{i=1}^{n} 1 = n$

7. [10 Points] Find the equation of the tangent line to $y = \cos(\ln(x+1)) + \ln(\cos x) + e^{\sin x} + \sin(e^x - 1)$ at the point where $x = 0$.

8. [20 Points] Let $\boldsymbol{f(x) = \frac{x^4}{e^x} = x^4 e^{-x}}$. For this function, discuss domain, vertical and horizontal asymptote(s), interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve.

Take my word that $\displaystyle\lim_{x\to\infty} f(x) = 0$ and $\displaystyle\lim_{x\to-\infty} f(x) = +\infty$.

Also take my word that $\displaystyle f'(x) = \frac{x^3(4-x)}{e^x}$ and $\displaystyle f''(x) = \frac{x^2(x-2)(x-6)}{e^x}$.

9. [15 Points] A conical tank, 14 feet across the entire top and 12 feet deep, is leaking water. The water is leaking at the rate of 2 cubic feet per minute. How fast is the radius of the water level changing when the radius of the water level is 3 feet? **Recall the volume of the cone is given by $V = \frac{1}{3}\pi r^2 h$

10. [15 Points] A cylindrical can, with a bottom and a top, has a fixed volume of 2000 π cubic units. Determine the height and radius of the can that has the least surface area. (Recall, the volume of a cylinder with radius r and height h, is given by $V = \pi r^2 h$.) (Remember to state the domain of the function you are computing extreme values for.)

11. [15 Points] Consider the region in the first quadrant bounded by $y = e^x + 1$, $y = 4$, and the y–axis. (a) Draw a picture of the region. (b) Compute the area of the region.

(c) Compute the volume of the three-dimensional solid obtained by rotating the region about the horizontal line $\boldsymbol{y = -2}$. Sketch the solid, along with one of the approximating washers.

12. [15 Points] Consider an object moving on the number line such that its velocity at time t seconds is $\boldsymbol{v(t) = 4 - t^2}$ feet per second. Also assume that the position of the object at one second is $\frac{5}{3}$.

(a) Compute the acceleration function $a(t)$ and the position function $s(t)$.

(b) Compute the **total distance** travelled for $0 \le t \le 3$.