Position–Velocity Worksheet

Answer Key

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Math 11

Read the following problems and answer each of the given questions:

SEE ME IF YOU HAVE QUESTIONS ABOUT THE DIAGRAMS

1. Suppose a falling ball's position is given by $s(t) = 256 - 16t^2$ feet at t seconds.

   • What is the ball's initial position above the ground?

   Initial position is $\boxed{s(0) = 256}$ feet.

   • Find the average velocity of the ball during the initial two seconds of its drop.

   Average velocity= $V_{\text{ave}} = \dfrac{\text{change in distance}}{\text{change in time}} = \dfrac{s(2) - s(0)}{2 - 0} = \dfrac{(256 - 16(2)^2) - 256}{2} = \dfrac{-64}{2} = \boxed{-32}\dfrac{\text{ft}}{\text{sec}}$

   • Find the velocity at 2 seconds and 3 seconds respectively.

   $v(t) = s'(t) = -32t$ so $\boxed{v(2) = -64}\frac{\text{ft}}{\text{sec}}$ and $\boxed{v(3) = -96}\frac{\text{ft}}{\text{sec}}$

   Understand why the velocity is negative here. The ball is falling in the negatively oriented direction. So the position $s(t)$ is decreasing $\searrow$ which means it's derivative $s'(t) = v(t)$ is negative.

   • How much time passed before the ball hit the ground?

   First find when the ball hits the ground. Set $s(t) = 0$ and solve for time t.

   $s(t) = 256 - 16t^2 = 0 \Longrightarrow t^2 = \frac{256}{16} \Longrightarrow t = \pm 4 \Longrightarrow \boxed{t = 4}$ seconds, since we are considernig positive time here. So the ball hits the ground in 4 seconds.

   • What was the ball's velocity when it hit the ground?

   The ball's velocity, when it hits the ground, is $v(4) = -32(4) = \boxed{-128}\frac{\text{ft}}{\text{sec}}$

   • Finally, find the ball's acceleration at 3 seconds.

   $a(t) = v'(t) = s''(t) = -32$ (note, it's constant here, acceleration due to gravity)

   So, $\boxed{a(3) = -32}\frac{\text{ft}}{\text{sec}^2}$

2. A ball is thrown straight upward from the ground with initial velocity $v_0 = 96$ feet per second. The height of the ball at time t is given by the position function $s(t) = -16t^2 + 96t$.

   - Find the maximum height attained.

   Maximum height is attained when $v(t) = 0$. Set $v(t) = -32t + 96 = 0$ and solve for time t. So, $t = \frac{96}{32} = 3$ seconds. That's only the time for the max height. We must plug $t = 3$ seconds into the position function to find the actual max height $= s(3) = -16(3)^2 + 96(3) = -144 + 288 = \boxed{144}$ feet.

   - Find the velocity with which the ball hits the ground upon its return.

   The ball hits the ground when $s(t) = 0 = -16t^2 + 96t = -16t(t-6)$. We can solve this equation for $t = 0$(start time) or $t = 6$ (time the ball impacts the ground).

   Then we plug this time $t = 6$ seconds into the velocity formula $v(t) = s'(t) = -32t + 96$. We have $v(6) = -32(6) + 96 = -192 + 96 = \boxed{-96 \frac{\text{ft}}{\text{sec}}}$. Take a second to think about why the velocity at impact is equal, but opposite in sign, to when the ball is launched. Think about slopes on the graph of the position function $s(t)$.

   - How much time past before the ball returned to the ground?

   As mentioned above, the ball hits the ground after $\boxed{6}$ seconds. It travels 3 seconds up, and 3 seconds down. Again, relate this to the symmetry of the graph for $s(t)$.

3. A squirrel is running along an East-West telephone wire with position at time t (in seconds) given by $s(t) = t^2 - 6t$ feet from a fixed telephone pole, with the positive direction being to the East. (A sketch might help.)

   - When is the squirrel moving East? West?

   Note that velocity is given by $v(t) = s'(t) = 2t - 6$. The squirrel is moving East, in the positively oriented direction, when $v(t) > 0$. That is, when $2t - 6 > 0$ or for time $\boxed{t > 3}$.

   The squirrel is moving West, in the negatively oriented direction, when $v(t) < 0$. That is, when $2t - 6 < 0$ or for time $\boxed{t < 3}$.

   - Where is the squirrel at time(s) when it changes direction?

   The squirrel changes direction when $v(t) = 0 = 2t - 6$. That is at $\boxed{t = 3}$ seconds.

   - What is the squirrel's total distance travelled from time $t = 0$ to $t = 8$?

   When the squirrel changes direction, the position at $t = 3$ seconds is $s(3) = 3^2 - 6(3) = 9 - 18 = -9$. So the squirrel travels 9 units West. Then it turns around and travels East for time $t > 3$. From time $t = 3$ to $t = 8$ seconds, the squirrel travels back 9 units to where it started, and then travels 16 more units further West from where it started. Note that $s(8) = 8^2 - 6(8) = 64 - 48 = 16$ in the positive direction.

   The total distance is given by $|s(0) - s(3)| + |s(3) - s(8)| = |0 - (-9)| + |-9 - 16| = 9 + 25 = \boxed{34}$.

- What is the squirrel's displacement(net distance) travelled from time $t = 0$ to $t = 8$?

The displacement (net distance) is given by $s(8) - s(0) = 16 - 0 = \boxed{16}$. The displacement doesn't give the nutcase squirrel credit for the distances of running back and forth, it just accounts for the final net gain in the (positive) direction.

4. A stone is dropped from a bridge that is 576 feet above a river. The stone's position is given in feet at time t by $s(t) = -16t^2 + 576$.

   - How long does it take for the stone to impact the water (fixed at position 0 here)?

   The stone impacts the water when $s(t) = 0 = -16t^2 + 576$. That is, when $t^2 = \dfrac{576}{16} = 36$. Thus, $t = \pm 6$, and we will only consider the positive time $t = 6$ seconds here. The stone impacts the water at $\boxed{t = 6}$ seconds.

   - What is the stone's velocity when it impacts the water?

   The velocity is given by $v(t) = s'(t) = -32t$. So the velocity of the stone when it impacts the water, at $t = 6$ seconds, is given by $v(6) = -32(6) = \boxed{-192 \frac{\text{ft}}{\text{sec}}}$.