Tips and a Systematic Procedure for Sketching Curves in Math 11

- **Domain:** Find the domain of the function $f(x)$.

- **Vertical Asymptotes:** Find the vertical asymptote(s). Recall, the line $x = a$ is a vertical asymptote if at least one of the following holds: $\lim_{x \to a^+} f(x) = \pm\infty$ or $\lim_{x \to a^-} f(x) = \pm\infty$.

- **Horizontal Asymptotes:** Find the horizontal asymptote(s). Recall, the line $y = L$ is a horizontal asymptote if either (or both) $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$.

- **First Derivative:** The first derivative will help you find critical numbers, which are where local extreme values possibly occur. Critical numbers are numbers (in the domain of the original function) where the first derivative is zero or undefined. The first derivative helps you decide on which interval(s) the graph is increasing or decreasing. Then the First Derivative Test helps you decide the location of local extreme values. Carefully use a sign-testing process (into the first derivative) to first analyze the sign of the derivative on the interval(s) of interest and then to decide where there are local extreme values (if any at all). Remember:
  - if $f'(x) > 0$ on an interval I, then $f(x)$ is increasing on I. That is, $f'(x)\oplus \Rightarrow f(x) \nearrow$.
  - if $f'(x) < 0$ on an interval I, then $f(x)$ is decreasing on I. That is, $f'(x)\ominus \Rightarrow f(x) \searrow$.

- **Second Derivative:** The second derivative will help you find inflection points where the concavity changes. Possible inflection points occur at numbers (in the domain of the original function) where $f''(x)$ equals zero or is undefined. The second derivative information helps you decide how the graph bends or turns, whether it's concave up or concave down. Carefully use a sign-testing process (into the second derivative), as well as the Concavity Test, to first analyze the sign of the second derivative on the interval(s) of interest and then to decide where there are actual inflection points. Remember:
  - if $f''(x) > 0$ on an interval I, then $f(x)$ is concave up on I. That is, $f''(x)\oplus \Rightarrow f(x) \bigcup$.
  - if $f''(x) < 0$ on an interval I, then $f(x)$ is concave down on I. That is, $f''(x)\ominus \Rightarrow f(x) \bigcap$.

- **Piece Together:** Piece the first and second derivative information together using our training from class and make a summary of the curve's general shape on each interval of interest.

- **Sketch:** Finally, plot a few specific points, like local mins, maxs, inflection points, and sketch and label the curve on the coordinate axes.

** Note: we can analyze even more information. We could investigate symmetry of the function in question. We could also investigate the left and right hand limits at the points where we have vertical asymptotes. Sometimes, looking at the x-intercepts or y-intercepts helps plot a few simple points. Anyhow, there is always more to do, but the above list is a pretty comprehensive list for gaining a nice understanding of curve sketching. Use our training to interpret what the derivative information tells us about the shape of the graph of a function.