Math 11-Benedetto

What you need to know for the Final Exam, since Exam #3

The exam will be weighted more heavily towards the material towards end of the semester, mainly because calculus builds on itself so well; but it will still cover the entire course. So you should know everything you needed to know for Exams 1, 2 and 3, **plus** you should know Sections 6.2, 7.3, and 7.4 The following is a list of most of the topics covered since Exam 3; it should be used together with the previous three review sheets to get a review for the entire course. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.**

- 6.2: Volumes of revolution. Given a description of a region in the plane, know how to set up an integral for the volume of the solid formed by rotating the region about any horizontal or vertical line. Know how to sketch both the 2D regions in the plane, as well as the 3D solids formed by rotation.
- 7.3: Logarithms. Know the definition (page 405) and basic properties (pages 405–406) of logarithms. Know the specific definition and properties of the natural logarithm $\ln x$ (pages 406–408).
- 7.4: Derivatives of Logarithms. Know the derivative of $\ln x$ (page 411) and the antiderivative of $1/x$ (page 414). Logarithmic Differentiation. You should be able to compute the derivative of x^x or $x^{\sin x}$.

Some Things You Don't Need to Know

- Volumes other than volumes of revolution (late in Section 6.2).
- Derivatives of general logarithmic functions (page 416). That is, you **do** need to know how to differentiate $\ln x$, but you **don't** need to know how to differentiate $\log_2 x$, for example.
- The number e as a limit (late in Section 7.4).
- Proofs of all theorems.

Tips

- Sections 7.2–7.4 mostly come down to knowing two main things: (1) the derivatives of e^x and $\ln x$, **and** (2) all the various relationships between exponential and logarithmic functions. (Like the fact that $e^{\ln x} = x$, that $\ln(xy) = \ln x + \ln y$, and all those (many) other facts.) Of course, know the other stuff like the basic properties of logarithms and exponentials, like their asymptotes and limits at ∞, too. See the overview handout on e^x and $\ln x$.

- Be familiar with computing integrals with exponentials and natural logs floating around. Hint: we do not know how to intergrate $\ln x$, so if your integral has a natural log piece in it, that piece might be a good choice for your u-substitution.

- Volumes of Revolution (Disc Method): Consider the region bounded by $y = f(x)$, the x-axis, $x = a$ and $x = b$. Rotating this region about the **x-axis** yields a 3–D solid. If we deal with vertical cross-sectional slices that are discs, we compute a volume

  $$V = \int_a^b \pi[f(x)]^2 \, dx$$

  Think about the cross-sectional disc slice having an area of $\pi(\text{radius of the slicing disc})^2 = \pi(f(x))^2$.

- Volumes of Revolution (Disc Method): Consider the region bounded by $x = g(y)$, the y-axis, $y = c$ and $y = d$. Rotating this region about the **y-axis** yields a 3–D solid. If we deal with horizontal cross-sectional slices that are discs, we compute a volume

  $$V = \int_c^d \pi[g(y)]^2 \, dy$$

  Think about the cross-sectional disc slice having an area of $\pi(\text{radius of the slicing disc})^2 = \pi(g(y))^2$.

- Volumes of Revolution (Washer Method): Consider the region bounded by $y = f(x)$ (on top), $y = g(x)$ (on bottom), $x = a$ and $x = b$. Rotating this region about the **x-axis** yields a 3–D solid. If we deal with vertical cross-sectional slices that are washers, we compute a volume

  $$V = \int_a^b \pi[(f(x))^2 - (g(x))^2] \, dx$$

  Think about the cross-sectional washer slice having an area of $\pi[(\text{outer radius})^2 - (\text{inner radius})^2] = \pi[(f(x))^2 - (g(x))^2]$.