Math 11, D. Benedetto

## What you need to know for Exam 3

You should know Sections 4.7, 4.9, all of Chapter 5, and Sections 6.1, and 7.2. The following is a list of most of the topics covered since Exam 2. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.**

- 4.7: Optimization. Given a word problem asking you to minimize or maximize a given quantity, be able to convert the problem to the problem of finding extreme values of a function on an interval.

- 4.9: Antiderivatives. Given a function $f$, know the definition of *an* antiderivative of $f$, and know the definition of *the* (most general) antiderivative of $f$. Know how to compute the most common antiderivatives that we have seen; they can be found in the table on page 325. Know the applications to rectilinear motion (pp. 277–279).

- 5.1: Areas and Distances. Know the idea of the Riemann sum as a way to approximate area or distance. Know how to compute $R_n$ (right endpoint) and $L_n$ (left endpoint) Riemann sums using $n$ equal-width intervals, but we mostly focussed on the right endpoint case. Sigma notation.

- 5.2: The Definite Integral. The definition of the definite integral as a limit of Riemann sums. Notation for the definite integral. Evaluating definite integrals using Sigma identities (equations 5–11, page 303) and limit laws. Properties of the definite integral (page 307, *all* of the boxed equations, as well as equation 5 on page 308; but *not* the comparison properties on page 309).

- 5.3: FTC. The Fundamental Theorem of Calculus, Parts I and II. Know the statements of both parts (they're stated together on page 320), and know how to use them to differentiate integral-defined functions (including strange looking problems like Example 4, page 317) and to compute definite integrals.

- 5.4: Indefinite Integrals. Know the definition of indefinite integrals, and don't confuse them with definite integrals. As mentioned above, know the table of indefinite integrals on page 325. Be able to evaluate some integrals, sometimes using algebra. Recognize the uses of the Net Change Theorem. Know how to use velocity to compute both displacement and total distance traveled (pages 327–329).

- 5.5: Substitution. Know the substitution rule for indefinite integrals (page 334) and for definite integrals (page 336). Practice **a lot** to make sure you actually can use them.

- 6.1: Areas between curves. Know how to sketch a region when told its bounding curves. We only integrated with respect to $x$. Be able to determine limits of integration (i.e., where the region starts and ends), as well as knowing which function is on top and which is on bottom.

- 7.2: Exponentials. Know the definition of $a^x$ for any positive $a > 0$ and any real number $x \in \mathbb{R}$. (See pages 392–393.) Know the basic properties of exponential functions (boxes 2 and 3, pages 394–395). Most importantly, know the definition of the number $e$ (page 397), the derivative of $e^x$ (page 398), the properties of $e^x$ (page 400, box 10), and the antiderivative of $e^x$ (page 401).

## Some Things You Don't Need to Know

- Riemann sums which use $x_i^*$ something besides the right or left endpoint.
- Computing certain specific integrals by interpreting them as areas we know, like Example 4b, page 305, something other than when your function is a line.
- The midpoint rule (page 306).
- Comparison properties of integrals (the box on page 309).
- Symmetry in integration (pages 337–338), although you are free to use it.
- Volumes of Revolution (Section 6.2)
- Logarithms (Section 7.3-7.4)
- Derivatives of general logarithmic and exponential functions (page 416). That is, you **do** need to know how to differentiate $e^x$ or $e^{u(x)}$, but you **don't** need to know how to differentiate $\log_2 x$ and $2^x$, for now, for example.
- The number $e$ as a limit (late in Section 7.4).
- Proofs of all theorems.

## Tips

- Know how to compute an integral from the limit definition (i.e., chop up the interval, make the Riemann sum, use Sigma identities, and take the limit).
- Remember, there are two ways to do a definite integral by substitution: either ignore the limits of integration and do it like an indefinite integral, back-substituting to$x$ from $u$ at the end, and then plugging in the original limits of integration, **or**, as we followed by the book's method, change the limits according to box 5, page 336 (the Substitution Rule for Definite Integrals). **Do not mix and match these two methods**— do one or the other. Also, when you change to the new variable $u$, you **must** change the limits accordingly; keeping the original limits of integration(without correct resubstitution will lose you some points.
- When you're doing an **indefinite integral**, you need the $+C$. When you're doing a **definite integral**, that is a number, you do **not** need to have the $+C$.

- Practice computing integrals. Remember, you have two main tools: you can simplify the integrand using algebra, and/or you can do a substitution. Those are basically your only choices in Math 11! The idea is to reduce the integrand to something you actually know the antiderivative of. The key thing is to **keep trying different ideas until something works**. (But if you really get stuck, don't waste your time; move on to another problem, and come back if you have time later.)

- Study more if, upon seeing an integral, you cannot **quickly** recognize which approach for integration to use. You should be able to immediately know what to do and quickly make the necessary computations. Each integral should take you about 1-3 minutes to compute. If you cannot work quickly, you're not ready for the exam. Ask me for help. I have some good ideas for approaching integrals. Practice a combined technique of speed and accuracy!!!

- Don't forget about all those motion problems: finding velocity $v(t)$ and position $s(t)$ given $a(t)$, and then answering any relevant position/velocity questions, as well as finding total distance travelled.

- For optimization and area **draw pictures**. And practice, practice. Don't forget to read the problem carefully: don't compute an area if you're asked for a volume, and don't use the wrong region in the plane. On optimization problems, make sure you specify your interval, and **don't use the closed interval method unless your interval is actually closed**.

- For optimization problems, after you translate the word problem into formulas, be careful to pay attention to which equation has fixed information and which you are trying to maximize or minimize. Often you will use the fixed information to solve for one variable in terms of the other, and then substitute into your other equation that you are finding extremum for. Then you will convert it to an equation in terms of one variable. (Don't forget to **explicitly** write down both the **function** and the **interval**.) Then use calculus to find the extremum. The closed interval method is sometimes useful, but not always; sometimes you need the First Derivative Test for Absolute Extrema. You should make sure and show that you do indeed have a maximum or a minumum, probably by using sign testing into the first derivative, around the critical number.