Extra Example for Related Rates

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A swimming pool is 40 feet wide, 100 feet long, has a slanted base that, when the pool is full, starts at water level and slopes down evenly to 12 feet at the opposite end, 100 feet away. The pool is being filled. When the water is 3 feet from the top, water is being pumped in at 60 ft^3 per second. How fast is the water level rising at that moment?

- Diagram

- Variables

Let x = length of the water level at time t

Let y = height of the water level at the deep end at time t

Let V = volume of the water in the pool at time t

Find $\frac{dy}{dt} =?$ when $y = 9$ feet (3 feet from the top)

and $\frac{dV}{dt} = 60\frac{\text{ft}^3}{\text{sec}}$

- Equation relating the variables:

Volume=Base $\cdot$ Height. We can use the side view of the pool for the base. First find the area of the triangle on the side and then multiple by the width of the pool (which is 40 feet here) to get the volume.

$V = \frac{1}{2}xy \cdot 40 = 20xy$

- Extra solvable information: Note that x is not mentioned in the problem's info. But there is a relationship, via similar triangles, between x and y. We must have

$$\frac{x}{100} = \frac{y}{12} \implies x = \frac{100y}{12} = \frac{25y}{3}$$

After substituting into our previous equation, we get:

$V = 20xy = 20\left(\frac{25y}{3}\right)y = \frac{500}{3}y^2$

- Differentiate both sides w.r.t. time t.

$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{500}{3}y^2\right) \implies \frac{dV}{dt} = \frac{500}{3} \cdot 2y \cdot \frac{dy}{dt} \implies \frac{dV}{dt} = \frac{1000}{3}y\frac{dy}{dt}$

- Substitute Key Moment Information (now and not before now!!!):

$60 = \frac{1000}{3}(9)\frac{dy}{dt}$

- Solve for the desired quantity:

$\frac{dy}{dt} = \frac{60}{3000} = \frac{1}{50}\frac{\text{ft}}{\text{sec}}$

- Answer the question that was asked: The water is rising at a rate of $\frac{1}{50}$ feet every second.