

Name: \_\_\_\_\_

**Amherst College**  
**DEPARTMENT OF MATHEMATICS**  
**Math 11**  
**Midterm Exam #3**  
**April 29, 2011**

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
  
- You need *not* simplify algebraically complicated answers for the derivative section. However, numerical answers such as  $\sin\left(\frac{\pi}{6}\right)$ ,  $4^{\frac{3}{2}}$ ,  $e^0$  should be simplified.
  
- Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		20
2		25
3		10
4		15
5		20
6		10
Total		100

**1.** [20 Points] **Differentiate** each of the following functions. You **do not** need to simplify your answers.

(a)  $f(x) = \int_{\tan x}^9 \sqrt{\sin t + 2t} dt$

(b)  $f(x) = e^{\frac{1}{x}} - \frac{1}{8e^x} + \frac{1}{8 + e^x}$

(c)  $f(x) = \sec(2 + e^x) + e^{(2 + \sec x)}$

(d)  $f(x) = e^x + x^e + e^e + e^{(e^x)} + (x^e)^e$ .

**2.** [25 Points] Compute each of the following integrals:

(a)  $\int \left( e^{7x} + \frac{1}{e^{4x}} \right)^2 dx$

(b)  $\int x(x-1)^{\frac{5}{7}} dx$

**2.** [Continued] Compute the following integral:

(c)  $\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \frac{\cos(2x)}{\sin^3(2x)} dx$

**3.** [10 Points] Find the function  $f(x)$  that satisfies  $f'(x) = \frac{e^x}{\sqrt{e^x + 3}}$  and  $f(0) = 7$ .

**4.** [15 Points] You need to construct a box with a square base for a fixed cost of \$96. The material for the bottom and top costs \$4 per square foot, and the material for the sides costs \$1 per square foot. What is the maximum volume possible for your box?

(Don't forget to state the common sense bounds.)

5. [20 Points] Use Riemann Sums and the limit definition of the definite integral to

$$\text{compute } \int_1^4 x^2 dx.$$

\*\*\* Recall  $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$ ,  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$ , and  $\sum_{i=1}^n 1 = n$

**6.** [10 Points] A moving object has velocity  $v(t) = t^2 - 3t + 2$  feet per second, at time  $t$  seconds. Compute the **Total Distance** travelled by this object from time  $t = 0$  to  $t = 3$  seconds.



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# OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1    Compute  $\int_0^3 \sqrt{9-x^2} \, dx$

OPTIONAL BONUS #2    Compute  $\lim_{n \rightarrow \infty} \frac{e^1 + e^{(1+\frac{2}{n})} + e^{(1+\frac{3}{n})} + \dots + e^2}{n}$