Math 11   Midterm Exam #3   Compact Version   December 3, 2010

● This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

● You need *not* simplify algebraically complicated answers for the derivative section. However, numerical answers such as $\sin\left(\frac{\pi}{6}\right)$, $4^{\frac{3}{2}}$, e^0, should be simplified.

● Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

1. [15 Points]   Answer each of the following:

(a) Differentiate $f(x) = \displaystyle\int_{\sin x}^{7} \sqrt{1 - e^t}\, dt$

(b) Differentiate $f(x) = e^{\left(e^x\right)} + \left(x^e\right)^e$

(c) Find $f(x)$ if $\displaystyle\int_{1}^{x} f(t)\, dt = e^{\frac{1}{x}} - e.$

2. [20 Points]   Compute each of the following integrals:

(a) $\displaystyle\int \left(e^{3x} + \frac{1}{e^x}\right)^2 dx$

(b) $\displaystyle\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \sec^2(3x)\, dx$

(c) $\displaystyle\int x(1-x)^{\frac{1}{3}}\, dx$

3. [8 Points]   Find the function $f(x)$ that satisfies $f'(x) = \dfrac{e^x + \cos x}{\sqrt{e^x + \sin x}}$ and $f(0) = 5.$

4. [15 Points]   You need to construct a soup can in the shape of a cylinder. The bottom of the can needs to be covered with three layers of material. The sides of the can need to be covered with two layers of material. The top only needs one layer. You are to use a fixed 1200π square cm of material. What is the maximum volume of your soup can?

(Don't forget to state the common sense bounds.)

5. [20 Points]   Compute $\displaystyle\int_1^4 6 - 3x \ dx$ using each of the following **three** different methods:

(a) Area interpretations of the definite integral,
(b) Fundamental Theorem of Calculus,
(c) Riemann Sums and the limit definition of the definite integral $***$.

$***$Recall $\displaystyle\sum_{i=1}^n i = \frac{n(n+1)}{2}$ and $\displaystyle\sum_{i=1}^n 1 = n$

6. [10 Points]   A stone is dropped from the top edge of a building. It hits the ground with a speed of 128 feet per second. What is the height of the building?

7. [12 Points]   A moving object has velocity $v(t) = t^2 - 3t + 2$ feet per second, at time t seconds. Compute the **Total Distance** travelled by this object from time $t = 0$ to $t = 3$ seconds.

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OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

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OPTIONAL BONUS #1   Use the **Limit Definition of the Derivative** to compute the derivative of $f(x) = \dfrac{e^x}{e^x + 1}$.

OPTIONAL BONUS #2   Compute $\displaystyle\int_0^6 \sqrt{6x - x^2} \ dx$

OPTIONAL BONUS #3   Compute $\displaystyle\int \sqrt{1 + \sqrt{x}} \ dx$

OPTIONAL BONUS #4   Compute $\displaystyle\int_{-3}^3 \tan x + \frac{\sqrt[3]{x}}{(1 + x^2)^7} - x^{17} \cos x \ dx$

OPTIONAL BONUS #5   Compute $\displaystyle\lim_{n \to \infty} \frac{\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{n}}{n\sqrt{n}}$