Math 11-Benedetto Midterm Exam #2 (Compact Version) March 26, 2010

1. [12 Points] Compute each of the following **limits**. Justify your answers.

(a) $\lim_{x\to 0} \dfrac{\sin(3x)}{x}$

(b) $\lim_{x\to 0} \dfrac{x^2 - x^3}{\cos x \cdot \sin^2(7x)}$

(c) $\lim_{x\to \infty} \dfrac{x^2 + 9}{x^7 - 4x + 7}$

(d) $\lim_{x\to \infty} \dfrac{19x^7 + 4x^5 - 8}{3x^7 + 2010x}$

2. [18 Points] **Differentiate** each of the following functions. You **do not** need to simplify your answers. Please do not waste time simplifying your derivative.

(a) $f(x) = \sqrt{\cos\left(\dfrac{1}{x}\right)}$

(b) $f(x) = (9 - x^2)^8 (x^3 - 6x)^9$

(c) $f(x) = \dfrac{1}{\left(\tan x + \dfrac{1}{x^2}\right)^{\frac{5}{7}}}$

3. [10 Points] Find the **absolute maximum** and **absolute minimum value(s)** of the function

$$G(x) = \frac{10x}{x^2 + 1} \quad \text{on the interval} \quad [0, 2].$$

4. [12 Points] Let $\boldsymbol{f(x) = x^3 - 3x + 3}$.

For this function, discuss domain, vertical and horizontal asymptote(s), interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve.

5. [18 Points] Let $\boldsymbol{f(x) = \dfrac{x^2 - 16}{x^2 - 9}}$.

For this function, discuss domain, vertical and horizontal asymptote(s), interval(s) of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve.

Take my word for it that (you do **not** have to compute these)

$$\boldsymbol{f'(x) = \frac{14x}{(x^2 - 9)^2}} \quad \text{and} \quad \boldsymbol{f''(x) = \frac{-42(x^2 + 3)}{(x^2 - 9)^3}}.$$

6. [7 Points] Find the **equation of the tangent line** to the curve $\boldsymbol{x^3 + x^2y + 4y^2 = 6}$ at the point $\boldsymbol{(1, 1)}$.

7. [15 Points] A conical tank, 14 feet across the entire top and 12 feet deep, is leaking water. The radius of the water level is decreasing at the rate of 2 feet per minute. How fast is the water leaking out of the tank when the radius of the water level is 2 feet?

**Recall the volume of the cone is given by $V = \frac{1}{3}\pi r^2 h$

8. [8 Points] A ball is thrown straight upward from the ground with initial velocity $v_0 = 96$ feet per second. The height of the ball at time t is given by the position function $s(t) = -16t^2 + 96t$.

• Find the maximum height attained by the ball.

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OPTIONAL BONUS

Do not attempt this unless you are completely done with the rest of the exam.

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OPTIONAL BONUS A kite 300 feet high is being blown horizontally at 10 feet per second. When the kite has blown horizontally for 40 seconds, how fast is the angle between the string and the horizontal changing?