Review Packet for Exam #2

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Math 11

Differentiation Rules Differentiate the following functions. Please do **not** simplify your derivatives here.

1. $y = \sin^3(x^3)$
2. $y = \cos^2(3x)$
3. $f(t) = t^2 \sin^5(2t)$
4. $H(x) = \left(1 - \frac{2}{x^2}\right)^5$
5. $f(x) = \sqrt[3]{x^3 + 8}$
6. $g(t) = \dfrac{t^3 + \tan\left(\frac{1}{t}\right)}{1 + t^2}$
7. $p(x) = \dfrac{1}{(-2x + 3)^5}$
8. $r(x) = \dfrac{(2x + 1)^3}{(3x + 1)^4}$
9. $S(x) = \left(\dfrac{1 + 2x}{1 + 3x}\right)^4$
10. $g(x) = \cos(3x)\sin(4x)$
11. $g(x) = \dfrac{\cos(3x)}{\sin(4x)}$
12. $g(x) = (x + 7x^{-6})\sqrt{2x + 1}\cos^2(6x)$
13. $w = \dfrac{5(1 + x^2)^3}{x\sqrt{2x + 1}}$
14. $y = \left((x^2 + 3x)^4 + x\right)^{-\frac{5}{7}}$
15. $g(t) = \cos\left(\sin^3\left(\dfrac{t}{\sqrt{t + 1}}\right)\right)$
16. $g(x) = \cos^2(6x)\left(\dfrac{\tan(-x)}{\sqrt{2x + 1}}\right)$

Absolute Extreme Values

17. Find the absolute maximum and absolute minimum values of the following functions on the given intervals.

    (a) $f(x) = 3x^{2/3} - \frac{x}{4}$ on $[-1, 1]$.

    (b) $h(x) = \frac{x^2 - 1}{x^2 + 1}$ on $[-1, 3]$.

    (c) $F(x) = -2x^3 + 3x^2$ on $[-\frac{1}{2}, 3]$.

    (d) $f(x) = x^{\frac{2}{3}}$ on $[-1, 8]$.

    (e) $f(x) = \frac{1}{1 + x^2}$ on $[-3, 1]$.

Curve Sketching For each of the following functions, discuss domain, vertical and horizontal asymptotes, intervals of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve.

18. $f(x) = x^3 - 3x^2 + 3x + 10$

19. $f(x) = \frac{3x^2}{1 - x^2}$

20. $f(x) = \frac{x}{x - 2}$

21. $f(x) = 2x^3 + 5x^2 - 4x$

22. $y = x^{\frac{1}{3}}$

23. $G(x) = -x^4$

24. $f(x) = 3x^4 + 4x^3$

25. $f(x) = x^4 - 6x^2$

26. $f(x) = \frac{3x^5 - 20x^3}{32}$

27. $f(x) = \frac{1}{x^2 - 9}$

28. $f(x) = \frac{2x^3 + 45x^2 + 315x + 600}{x^3}$. Take my word for it that (you do NOT have to compute these) $f'(x) = \frac{-45(x + 4)(x + 10)}{x^4}$ and $f''(x) = \frac{90(x + 5)(x + 16)}{x^5}$.

More derivatives

29. Let f and g be two differentiable functions, and suppose that their values and the values of their derivatives at $x = 1, 2, 3$ are given by the following table:

x	1	2	3
$f(x)$	3	2	5
$f'(x)$	-2	1	3
$g(x)$	3	1	4
$g'(x)$	-3	2	7

Let $h(x) = f \circ g(x)$ and $k(x) = f(x) \cdot g(f(x))$. Compute $h'(2)$ and $k'(1)$.

30. Let f and g be two differentiable functions, and suppose that their values and the values of their derivatives at $x = 2, 3$ are given by the following table:

x	2	3
$f(x)$	4	0
$f'(x)$	1	-7
$g(x)$	3	-1
$g'(x)$	-5	4

Let $h(x) = f(x)g(x), k(x) = \dfrac{f(x)}{g(x)}$ and $W(x) = f \circ g(x)$. Compute $h'(2)$ and $k'(2)$ and $W'(2)$.

31. Let $f(x) = \sqrt{x+1} \cdot g(x)$ where $g(0) = -7$ and $g'(0) = 4$. Compute $f'(0)$.

32. Let $f(x) = \dfrac{\sqrt{x^2+1}}{g(x)}$ where $g(0) = -7$ and $g'(0) = 4$. Compute $f'(0)$.

More Tangent Lines

33. For each of the plane curves described below, find an equation of the tangent line to the curve at the given point.

(a) $x^3 + x^2y + 4y^2 = 6$ at $(1, 1)$.

(b) $4(x + y)^2 = x^2y^2$ at $(-2, 1)$.

(c) $\dfrac{x}{y+1} = x^2 - y^2$ at $(1, 0)$.

(d) $4 \cos x \sin y = 3$ at $(\pi/6, \pi/3)$.

(e) $y^3 - xy^2 + \cos(xy) = 2$ at $(0, 1)$

Limits Evaluate the following limits. Please show your work.

34. $\lim_{x \to 0} \frac{\sin(5x)}{2x} =$

35. $\lim_{w \to 0} \frac{\sin(16w)}{w} =$

36. $\lim_{x \to 0} \frac{\sin(3x)}{\sin(2x)} =$

37. $\lim_{\theta \to 0} \frac{\sin^2(\theta)}{\theta^2 + 5\theta^3} =$

38. $\lim_{x \to 0} \frac{2x}{\sin(3x)} =$

39. $\lim_{h \to 0} \frac{\tan(6h)}{7h} =$

40. $\lim_{x \to 0} \frac{x + x\cos x}{\sin x \cos x} =$

41. $\lim_{x \to 0} \frac{\tan(5x)}{\sin(5x)} =$

42. $\lim_{x \to -\infty} \frac{x^3 - 2x}{4x^3 + 1}$

43. $\lim_{x \to \infty} \frac{x^3 + 1}{x^7 + 2x^{7/2}}$

44. $\lim_{x \to \infty} \frac{x^6 + 1}{x^3 + 9x^2 + 7}$

45. $\lim_{x \to 0} \frac{\sin^2 x}{4x^2 + 5x^3}$

46. $\lim_{x \to 0} \frac{x^2 \cos x}{x + 1}$

Related Rates

47. A conical reservoir, 12 ft. deep and also 12 ft. across the top is being filled with water at the rate of 5 cubic feet per minute. How fast is the water rising when it is 4 feet deep?

48. A kite 100 feet high is being blown horizontally at 8 feet per second. When there are 300 feet of string out, (a) how fast is the string running out? (b) how fast is the angle between the string and the horizontal changing?

49. Suppose a snowball remains spherical while it melts with the radius shrinking at one inch per hour. How fast is the volume of the snowball decreasing when the radius is 2 inches?

50. A point moves around the circle $x^2+y^2=9$. When the point is at $(-\sqrt{3}, \sqrt{6})$, its x-coordinate is increasing at the rate of 20 units per second. How fast is its y-coordinate changing at this instant?

51. A waterskier skis up over the ramp at a speed of 30 ft./sec. The 100 ft. ramp slopes straight from no height at one end to 20 feet on the other end. How fast is she rising vertically just as she leaves the ramp?

52. A cylindrical reservoir, 12 feet across the top, is being filled with water at the rate of 5 cubic feet per minute. How fast is the water rising when it is 4 feet deep?

53. A photographer is televising a 100-yard dash from a position 5 yards from the track in line with the finish line. When the runners are 12 yards from the finish line, the camera is turning at the rate of $\frac{3}{5}$ radians per second. How fast are the runners moving then?

54. Two trucks leave a depot at the same time. Truck A travels east at 40 miles per hour, while Truck B travels north at 30 miles per hour. How fast is the distance between the trucks changing 60 minutes after leaving the depot?

55. Suppose a spherical balloon is inflated at the rate of 10 cubic inches per minute. How fast is the radius of the balloon increasing when the radius is 5 inches?

56. Suppose a 20 foot ladder is sliding down a vertical wall. Let θ be the angle formed by the ground and the base of the ladder. At what rate is the angle θ changing when the base of the ladder is sliding away from the wall at 2 feet per second and $\theta = \frac{\pi}{3}$?

57. Suppose a 50 foot ladder is sliding down a vertical wall. Let θ be the angle formed by the ground and the base of the ladder. At what rate is the angle θ changing when the base of the ladder is sliding away from the wall at 2 feet per second and the base of the ladder has slid 30 feet from the wall?

58. A hot circular plate of metal is cooling. As it cools its radius is decreasing at the rate of 0.01 cm/min. At what rate is the plate's area decreasing when the radius equals 50 cm?

59. A child riding in a car driving along a straight road is looking through binoculars when she sees a water tower off to the side. The tower is located 1500 ft from the nearest point on the road. At a particular moment, the car is moving at 80 feet per second, and the car is 800 feet from that nearest point to the tower. How fast must the child be rotating the angle that the binoculars are pointing to keep the tower in view?

60. A 6 foot tall man walks with a speed of 8 feet per second away from a street light that is atop an 18 foot pole. How fast is the top of his shadow moving along the ground when he is 100 feet from the light pole?

Position, Velocity, Acceleration

1. Suppose that Dan throws a ball, from the ground, straight upward in the air with an initial velocity of 128 meters per second. The ball reaches a height of $\mathbf{s(t) = 128t - 16t^2}$ feet in t seconds. Suppose Sam is lying on the ground under the ball. Answer the following questions:

(a) What is the maximum height the ball reaches?
(b) What is the ball's velocity at time $t = 5$?
(c) What is the ball's acceleration at time $t = 5$?
(d) At what time will the ball hit Sam?
(e) What is the ball's velocity when it hits Sam?
(f) What is the ball's acceleration when it hits Sam?

2. RETALIATION! When Dan saw that the ball actually hit Sam, he ran away, up a tree. Dan climbed up the tree exactly 155 feet (above the ground). Revenge was necessary! Sam managed to throw the ball upward at Dan with an initial velocity of 96 feet per second. This time the ball reaches a height of $\mathbf{s(t) = 96t - 16t^2}$ feet in t seconds.

Does the ball hit Dan? If it doesn't, explain why. If it does, explain why. Show your work.