Amherst College

DEPARTMENT OF MATHEMATICS

Math 11

Midterm Exam #1

February 18, 2011

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

• Please *show* all of your work and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Problem	Score	Possible Points
1		30
2		13
3		15
4		10
5		12
6		20
Total		100

1. [30 Points] Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(a) $\displaystyle\lim_{x\to-3} \frac{x^2-2x-15}{x^2+x-6} =$

(b) $\displaystyle\lim_{x\to5} \frac{x^2-2x-15}{|5-x|} =$

(c) $\displaystyle\lim_{x\to2} \frac{x^2-2x-15}{x^2+x-6} =$

1. (Continued) Evaluate each of the following limits. Please **justify** your answers. Be clear if the limit equals a value, $+\infty$ or $-\infty$, or Does Not Exist.

(d) $\displaystyle\lim_{x \to 5} \ \frac{x^2 - 2x - 15}{x^2 + x - 6} =$

(e) $\displaystyle\lim_{x \to 2} \ \frac{x + 7}{(x - 2)^2} =$

(f) $\displaystyle\lim_{x \to -1} \ \frac{H(x + 1) - H(-1 - x)}{x + 1} = \qquad$ where $H(x) = \sqrt{x + 2}$

2. [13 Points]   Prove that $\lim\limits_{x \to 5} 7 - 2x = -3$ using the $\varepsilon - \delta$ definition of the limit.

3. [15 Points]   Suppose that $f(x) = \dfrac{x+7}{x-3}$.   Compute $f'(x)$ using the **limit definition of the derivative**.

4. [10 Points] Suppose that $f(x) = x^3 + 7x^2 - 4x + 9$. Write the **equation of the tangent line** to the curve $y = f(x)$ when $x = -1$.

Use the limit definition of the derivative when computing the derivative.

5. [12 Points] Suppose that f and g are functions, **and**

- $\lim\limits_{x \to 7} f(x) = 5$
- $\lim\limits_{x \to 7} g(x) = -3$
- $f(5) = 7$
- $g(x)$ is continuous at $x = 7$.

Evaluate the following quantities and fully **justify** your answers. Do not just put down a value:

(a) $\lim\limits_{x \to 7} \sqrt{3f(x) - 7g(x)} =$

(b) $\lim\limits_{x \to 7} \dfrac{f(x)}{1 - x} =$

(c) $g(7) =$

(d) $g \circ f(5) =$

6. [20 Points] Consider the function defined by

$$f(x) = \begin{cases} \sqrt{x-3} & \text{if } x > 3 \\[2mm] 1 & \text{if } x = 3 \\[2mm] 6 - 2x & \text{if } 0 < x < 3 \\[2mm] 16 - x^2 & \text{if } -4 < x \leq 0 \\[2mm] \dfrac{1}{x+4} & \text{if } x < -4 \end{cases}$$

(a) Carefully sketch the graph of $f(x)$.

(b) State the **Domain** of the function $f(x)$.

6. (Continued) Continue to consider the function defined by

$$f(x) = \begin{cases} \sqrt{x-3} & \text{if } x > 3 \\[2mm] 1 & \text{if } x = 3 \\[2mm] 6 - 2x & \text{if } 0 < x < 3 \\[2mm] 16 - x^2 & \text{if } -4 < x \le 0 \\[2mm] \dfrac{1}{x+4} & \text{if } x < -4 \end{cases}$$

(c) Compute $\begin{cases} \displaystyle\lim_{x \to 0^+} f(x) = \\[3mm] \displaystyle\lim_{x \to 0^-} f(x) = \\[3mm] \displaystyle\lim_{x \to 0} f(x) = \end{cases}$

(d) Compute $\begin{cases} \displaystyle\lim_{x \to 3^+} f(x) = \\[3mm] \displaystyle\lim_{x \to 3^-} f(x) = \\[3mm] \displaystyle\lim_{x \to 3} f(x) = \end{cases}$

(e) Compute $\begin{cases} \displaystyle\lim_{x \to -4^+} f(x) = \\[3mm] \displaystyle\lim_{x \to -4^-} f(x) = \\[3mm] \displaystyle\lim_{x \to -4} f(x) = \end{cases}$

(f) State the value(s) at which f is discontinuous. Justify your answer(s) using definitions or theorems discussed in class.

OPTIONAL BONUS

Do not attempt these unless you are completely done with the rest of the exam.

OPTIONAL BONUS #1 Compute $\displaystyle\lim_{x \to 1} \frac{x^{\frac{2}{3}} - 1}{x^{\frac{1}{3}} - 1}$

OPTIONAL BONUS #2 Compute $\displaystyle\lim_{x \to 1} \frac{x^{\frac{1}{2}} - 1}{x^{\frac{1}{3}} - 1}$