Professor Danielle Benedetto

Math 11

Sketches for the piecewise defined functions from the Exam #1 review problems

Consider each of the following piecewise defined functions. Answer the related questions. *Justify* your answers please.

21. $\cancel{\times}$ Let $f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 \le x \le 2 \\ 8 - x & \text{if } x > 2 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

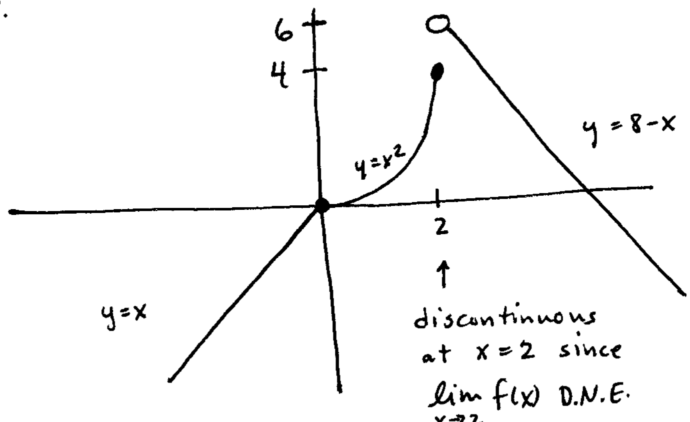
$\lim_{x\to 2} f(x) =$ DOES NOT EXIST since RHL $\neq$ LHL

$$\begin{cases} \text{LHL}: \lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2 = 4 \\ \text{RHL}: \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 8 - x = 6 \end{cases}$$

$\lim_{x\to 0} f(x) = 0$ since RHL= LHL

$$\begin{cases} \text{LHL}: \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x = 0 \\ \text{RHL}: \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 = 0 \end{cases}$$

Despite the fact that $f(2) = 4$ is defined, f is discontinuous at $x = 2$ since $\lim_{x\to 2} f(x)$ DOES NOT EXIST.



22. $\cancel{\times}$ Let $f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ 2x^2 & \text{if } 0 \le x \le 1 \\ 3 - x & \text{if } x > 1 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

$\lim_{x\to 2} f(x) = \lim_{x\to 2} 3 - x = 1$

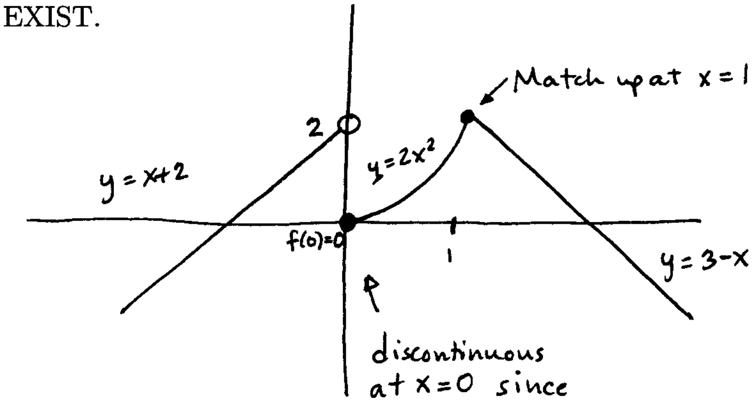
$\lim_{x\to 1} f(x) = 2$ since RHL= LHL

$$\begin{cases} \text{LHL}: \lim_{x\to^-} f(x) = \lim_{x\to 1^-} 2x^2 = 2 \\ \text{RHL}: \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 3 - x = 2 \end{cases}$$

$\lim_{x\to 0} f(x) =$ DOES NOT EXIST since RHL $\neq$ LHL

$$\begin{cases} \text{LHL}: \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x+2 = 2 \\ \text{RHL}: \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 2x^2 = 0 \end{cases}$$

Despite the fact that $f(0) = 0$ is defined, f is discontinuous at $x = 0$ since $\lim_{x\to 0} f(x)$ DOES NOT EXIST.



Note: $\lim_{x\to 1} f(x) = 2 = f(1)$ continuous !!! at $x=1$

23. Let $f(x) = \begin{cases} \dfrac{1}{x-4} & \text{if } x < 2 \\ \dfrac{1}{x} & \text{if } x \geq 2 \end{cases}$

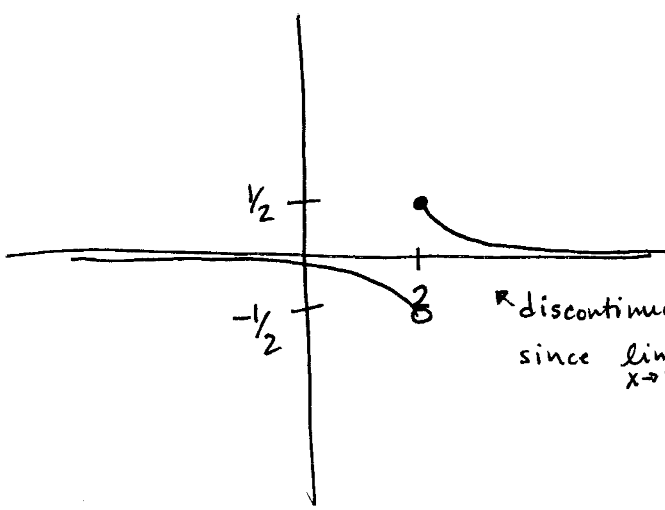
Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

$\lim_{x\to 1} f(x) = \lim_{x\to 1} \dfrac{1}{x-4} = -\dfrac{1}{3}$

$\lim_{x\to 2} f(x) =$ DOES NOT EXIST since RHL $\neq$ LHL

$$\begin{cases} \text{LHL}: \lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \dfrac{1}{x-4} = -\dfrac{1}{2} \\ \text{RHL}: \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} = \dfrac{1}{x} = \dfrac{1}{2} \end{cases}$$

Despite the fact that $f(2) = \dfrac{1}{2}$ is defined, f is discontinuous at $x = 2$ since $\lim_{x\to 2} f(x)$ DOES NOT EXIST.



24. Let $f(x) = \begin{cases} -3x+4 & \text{if } x \le 3 \\ -2 & \text{if } x > 3 \end{cases}$

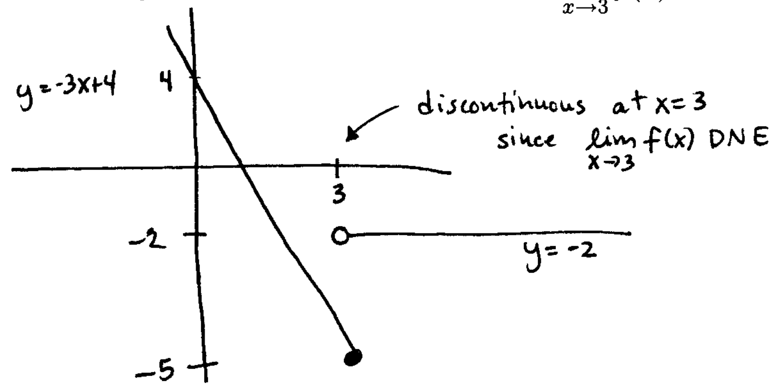
Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

$\lim_{x \to 3} f(x) =$ DOES NOT EXIST since RHL $\neq$ LHL.

$$\begin{cases} \text{LHL}: \lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} -3x+4 = -5 \\ \text{RHL}: \lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} -2 = -2 \end{cases}$$

$\lim_{x \to -2} f(x) = \lim_{x \to -2} -3x+4 = 10$

Despite the fact that $f(3) = -5$ is defined, f is discontinuous at $x = 3$ since $\lim_{x \to 3} f(x)$ DOES NOT EXIST.



25 Let $f(t) = \begin{cases} t-3 & \text{if } t \le 3 \\ 3-t & \text{if } 3 < t < 5 \\ 1 & \text{if } t = 5 \\ 3-t & \text{if } t > 5 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

$\lim_{t \to 3} f(t) = 0$ since RHL $=$ LHL

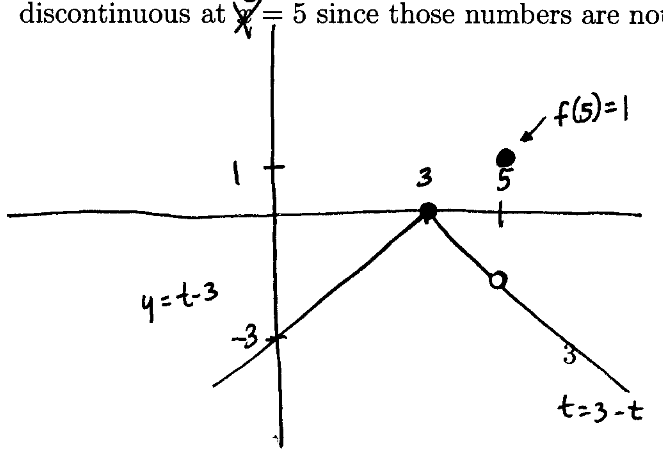
$$\begin{cases} \text{LHL}: \lim_{t \to 3^-} f(t) = \lim_{t \to 3^-} t-3 = 0 \\ \text{RHL}: \lim_{t \to 3^+} f(t) = \lim_{t \to 3^+} t-3 = 0 \end{cases}$$

$\lim_{t \to 0} f(t) = \lim_{t \to 0} t-3 = -3$

$\lim_{t \to 5} f(t) = -2$ since RHL=LHL

$$\begin{cases} \text{LHL}: \lim_{t \to 5^-} f(t) = \lim_{t \to 5^-} 3-t = -2 \\ \text{RHL}: \lim_{t \to 5^+} f(t) = \lim_{t \to 5^+} 3-t = -2 \end{cases}$$

Despite the fact that $f(5) = 1$ is defined, and $\lim_{x \to 5} f(x)$ exists and is equal to -2, f is discontinuous at $\cancel{x}\, t = 5$ since those numbers are not equal. That is, $\lim_{\cancel{x}\, t \to 5} f(\cancel{x}\, t) \neq f(5)$.

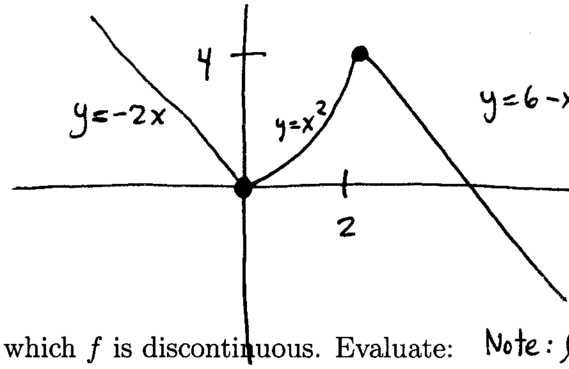


discontinuous at $t=5$ since

$\lim_{t \to 5} f(t) \neq f(5)$

Remember definition of continuity!!

26.



26. ~~K~~. Let $f(x) = \begin{cases} -2x & \text{if } x < 0 \\ x^2 & \text{if } 0 \le x \le 2 \\ 6-x & \text{if } x > 2 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

Note: $\lim_{x\to 0} f(x) = 0 = f(0)$ and $\lim_{x\to 2} f(x) = 4 = f(2)$ so continuous at both $x=0$ and $x=2$.

$\lim_{x\to -2} f(x) = \lim_{x\to -2} -2x = 4$

$\lim_{x\to 0} f(x) = 0$ since RHL=LHL

$$\begin{cases} \text{LHL}: \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} -2x = 0 \\ \text{RHL}: \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 = 0 \end{cases}$$

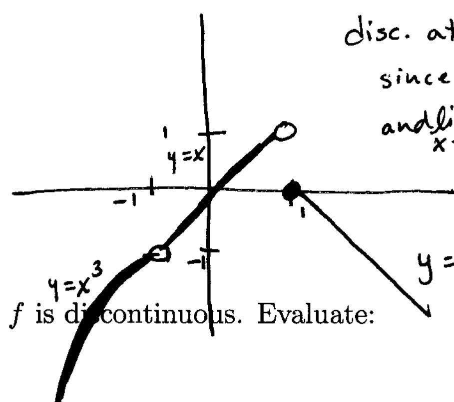
$\lim_{x\to 2} f(x) = 4$ since RHL= LHL

$$\begin{cases} \text{LHL}: \lim_{x\to 2-} f(x) = \lim_{x\to 2^-} x^2 = 4 \\ \text{RHL}: \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 6-x = 4 \end{cases}$$

$\lim_{x\to 6} f(x) = \lim_{x\to 6} 6 - x = 0$

Notice that f is continuous at all real numbers since the three pieces of the graph of the curve match up at the break points. Specifically $\lim_{x\to a} f(x) = f(a)$ for every number $x = a$.

27.



disc. at $x=-1$ and $x=1$ since $f(-1)$ undefined and $\lim_{x\to 1} f(x)$ DNE.

27. ~~K~~ Let $f(x) = \begin{cases} x^3 & \text{if } x < -1 \\ x & \text{if } -1 < x < 1 \\ 1-x & \text{if } x \ge 1 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

$\lim_{x\to -1} f(x) = -1$ since RHL= LHL

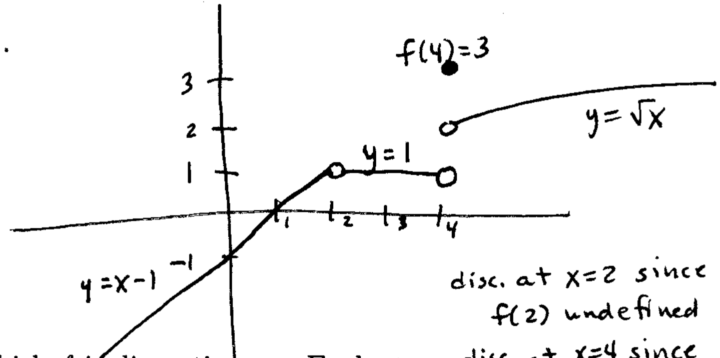
$$\begin{cases} \text{LHL}: \lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} x^3 = -1 \\ \text{RHL}: \lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} x = -1 \end{cases}$$

$\lim_{x\to 1} f(x) =$ DOES NOT EXIST since RHL $\neq$ LHL

$$\begin{cases} \text{LHL}: \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x = 1 \\ \text{RHL}: \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 1 - x = 0 \end{cases}$$

Despite the fact that $f(1) = 0$ is defined, f is discontinuous at $x = 1$ since $\lim_{x\to 1} f(x)$ DOES NOT EXIST. Also, f is discontinuous at $x=-1$ since $f(-1)$ undefined.

28.



disc. at $x=2$ since $f(2)$ undefined

disc. at $x=4$ since $\lim_{x\to 4} f(x)$ DNE

28. Let $f(x) = \begin{cases} x-1 & \text{if } x<2 \\ 1 & \text{if } 2<x<4 \\ 3 & \text{if } x=4 \\ \sqrt{x} & \text{if } x>4 \end{cases}$

Sketch the graph. Find the numbers at which f is discontinuous. Evaluate:

$\lim_{x\to 0} f(x) = \lim_{x\to 0} x-1 = -1$

$\lim_{x\to 2} f(x) = 1$ since RHL= LHL

$$\begin{cases} \text{LHL}: \lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x-1 = 1 \\ \text{RHL}: \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 1 = 1 \end{cases}$$

$\lim_{x\to 4} f(x) =$ DOES NOT EXIST since RHL$\neq$LHL

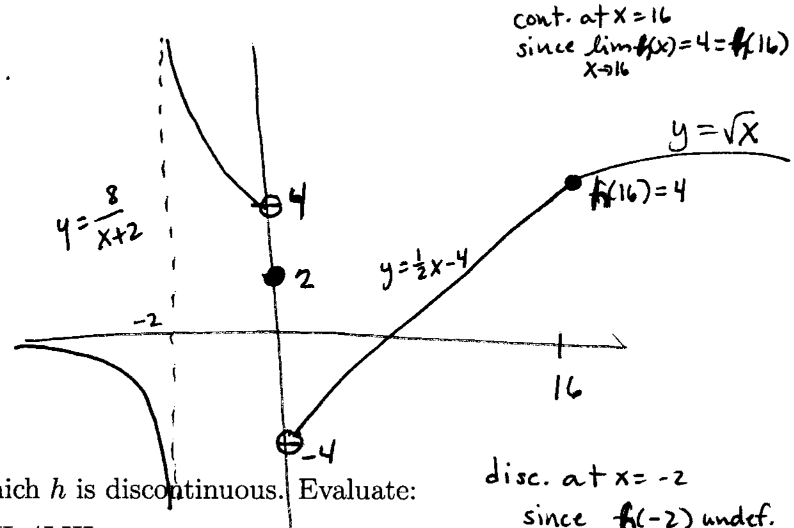
$$\begin{cases} \text{LHL}: \lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} 1 = 1 \\ \text{RHL}: \lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \sqrt{x} = 2 \end{cases}$$

$f(4) = 3$

Despite the fact that $\lim_{x\to 2} f(x)$ exists and is equal to 1, $f(2)$ is undefined. Thus, f is discontinuous at $x=2$. Also, despite the fact that $f(4)=3$ is defined, f is discontinuous at $x=4$ since $\lim_{x\to 4} f(x)$ DOES NOT EXIST.

cont. at $x=16$ since $\lim_{x\to 16} f(x) = 4 = f(16)$

29.



disc. at $x=-2$ since $f(-2)$ undef.

disc. at $x=0$ since $\lim_{x\to 0} h(x)$ DNE

29. Let $h(x) = \begin{cases} \dfrac{8}{x+2} & \text{if } x<0 \\ 2 & \text{if } x=0 \\ \dfrac{1}{2}x-4 & \text{if } 0<x\leq 16 \\ \sqrt{x} & \text{if } x>16 \end{cases}$

Sketch the graph. Find the numbers at which h is discontinuous. Evaluate:

$\lim_{x\to -2} h(x) =$ DOES NOT EXIST since RHL$\neq$LHL

$$\begin{cases} \text{LHL}: \lim_{x\to -2^-} h(x) = \lim_{x\to -2^-} \dfrac{8}{x+2} = -\infty \\ \text{RHL}: \lim_{x\to -2^+} h(x) = \lim_{x\to -2^+} \dfrac{8}{x+2} = +\infty \end{cases}$$

$\lim_{x\to 0} h(x) =$ DOES NOT EXIST since RHL$\neq$LHL

$$\begin{cases} \text{LHL}: \lim\limits_{x\to 0^-} h(x) = \lim\limits_{x\to 0^-} \dfrac{8}{x+2} = 4 \\ \text{RHL}: \lim\limits_{x\to 0^+} h(x) = \lim\limits_{x\to 0^+} \dfrac{1}{2}x - 4 = -4 \end{cases}$$

$\lim\limits_{x\to 16} h(x) = 4$ since RHL=LHL

$$\begin{cases} \text{LHL}: \lim\limits_{x\to 16^-} h(x) = \lim\limits_{x\to 16^-} \dfrac{1}{2}x - 4 = 4 \\ \text{RHL}: \lim\limits_{x\to 16^+} h(x) = \lim\limits_{x\to 16^+} \sqrt{x} = 4 \end{cases}$$

Note that h is discontinuous at $x = -2$ since h is undefined there, as well as the fact that $\lim\limits_{x\to -2} h(x)$ DOES NOT EXIST. Also, despite the fact that $h(0) = 2$ is defined, h is discontinuous at $x = 0$ since $\lim\limits_{x\to 0} h(x)$ DOES NOT EXIST.

30. ~~10.~~ Let $h(x) = \begin{cases} \dfrac{8}{x-2} & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ \dfrac{1}{2}x - 4 & \text{if } x > 0 \end{cases}$

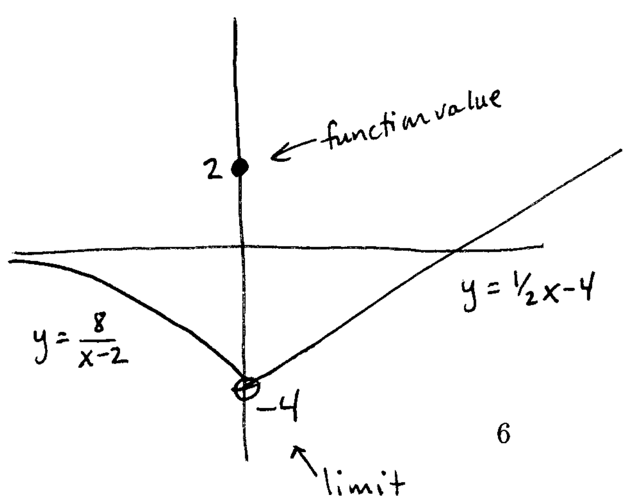
Sketch the graph. Find the numbers at which h is discontinuous. Evaluate:

$\lim\limits_{x\to 0} h(x) = -4$ since RHL=LHL

$$\begin{cases} \text{LHL}: \lim\limits_{x\to 0^-} h(x) = \lim\limits_{x\to 0^-} \dfrac{8}{x-2} = -4 \\ \text{RHL}: \lim\limits_{x\to 0^+} h(x) = \lim\limits_{x\to 0^+} \dfrac{1}{2}x - 4 = -4 \end{cases}$$

$\lim\limits_{x\to 2} h(x) = \lim\limits_{x\to 2} \dfrac{1}{2}x - 4 = -3$

Despite the fact that $h(0) = 2$ is defined, and $\lim\limits_{x\to 0} h(x)$ exists and is equal to -4, f is discontinuous at $x = 0$ since those numbers are not equal. That is, $\lim\limits_{x\to 0} h(x) \neq h(0)$.



note: $\lim\limits_{x\to 0} h(x)$ exists but doesn't match up with $h(0) = 2$.

31. Let $h(x) = \begin{cases} \dfrac{8}{x-2} & \text{if } x \le 0 \\ \dfrac{1}{2}x - 4 & \text{if } x > 0 \end{cases}$

Sketch the graph. Find the numbers at which h is discontinuous. Evaluate:

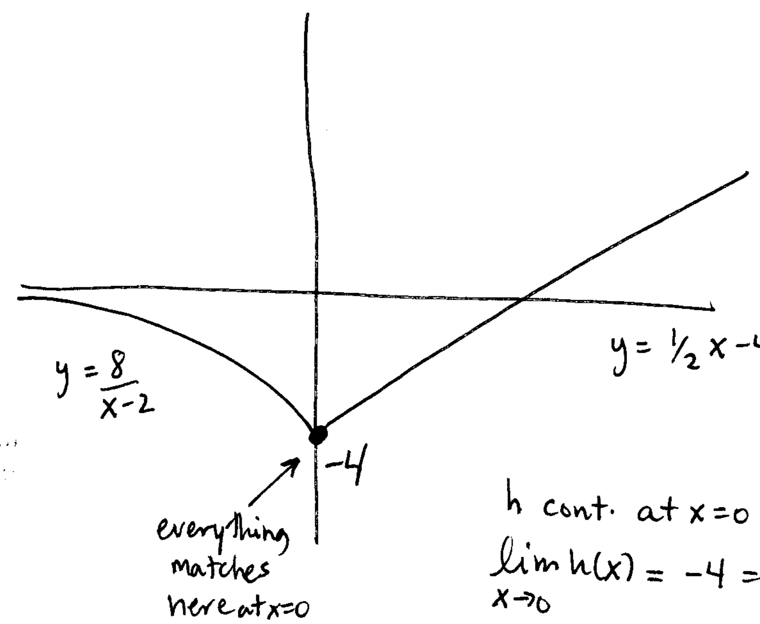
$\lim_{x\to 0} h(x) = -4$ since RHL=LHL

$$\begin{cases} \text{LHL}: \lim_{x\to 0^-} h(x) = \lim_{x\to 0^-} \dfrac{8}{x-2} = -4 \\ \text{RHL}: \lim_{x\to 0^+} h(x) = \lim_{x\to 0^+} \dfrac{1}{2}x - 4 = -4 \end{cases}$$

$\lim_{x\to 1} h(x) = \lim_{x\to 1} \dfrac{1}{2}x - 4 = -\dfrac{7}{2}$

$h(0) = -4$

Unlike the previous (similar) example, h is NO LONGER discontinuous at $x = 0$ since $h(0) = -4$ is defined, and $\lim_{x\to 0} h(x)$ exists and is equal to -4. Since those numbers are now equal, that is, $\lim_{x\to 0} h(x) = h(0)$, then h is now continuous at $x = 0$.



h cont. at $x=0$ since $\lim_{x\to 0} h(x) = -4 = h(0)$

[we "filled" the hole from the previous example.]