Math 106, Benedetto

What you need to know for Exam 3

This Exam covers Chapter 6 on the Natural Exponential and Logarithm. We studied their graphs, domains and ranges, algebra, values, as well as their relevant Limits, Derivatives and Integrals. Along the way, we took care to organize their Chain Rules and Antiderivatives, including the k-rule. For Integration, we paid particular attention to the three typical strategies: elementary antiderivative, algebra or u-substitution. We also covered Area between curves in Section 5.1. The following is a list of most of the topics covered. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.** Remember, no calculators in any exams.

- 6.2: Exponentials. Know the definition of a^x for any positive $a > 0$. Know the basic properties of exponential functions. Most importantly, know the definition of the number e, the derivative of e^x, Chain Rule, the algebra and properties of e^x, and the antiderivative of e^x and e^{kx} where k is a constant, what we called the k-rule. Key value: $e^0 = 1$.

- 6.1 Inverses. Know what 1-1 functions are. Know the definition of the Inverse function. Know how to find the graph of Inverse functions.

- 6.3: Logarithms. Know the definition, algebra and basic properties of the Natural Logarithms $\ln x$. Key value: $\ln 1 = 0$ but $\ln 0$ is **undefined**. Know the inverse properties $e^{\ln x} = x$ and $\ln e^x = x$. Practice simplifying $e^{-2\ln 3}$ or $\ln e^{-7}$.

- 6.4: Derivatives of Logarithms. Know the derivative of $\ln x$ and the antiderivative $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$. Remember the Absolute Value and $+C$. Know the Chain Rule too. Know how to use the Algebraic Properties of the Log to compute complex Log Derivatives. Also, Logarithmic Differentiation. You should be able to compute the derivative of, say, x^x or $x^{\sin x}$. Note: we do not know the antiderivative of $\ln x$.

- 5.1: Areas between curves. Know how to sketch a region when told its bounding curves. We only integrated with respect to x. Be able to determine limits of integration (i.e., where the region starts and ends), also determine where the functions intersect, as well as knowing which function is on top and which is on bottom.

  We will still be relying on u-substitution.

- 4.5: Integration using u-Substitution. Know the substitution rule for indefinite integrals and for definite integrals. Practice **a lot** to make sure you can use them. Make sure to **change** (or mark) your limits of integration in u-substitution. Here if $\ln x$ is present, it is likely used in your choice for u.

Techniques of Integration

1. "We Know It" Snap facts, MAKE A NEW LONGER LIST
2. Algebra
3. u-substitution

Common Types of Problems to Prepare, Know how to ...

- How to Graph e^x and $\ln x$. Also some key values. Be efficient here.
- How to compute derivatives, including Exponential rule, and Chain Rule. Know when it is a Power rule, an Exponential rule or a Constant Derivative. Know when the function has a variable or a constant formula. Note: e^x is a function and e^5 is a constant value. Make sure to use same Algebra Prep as before.
- How to use Algebra rules for e^x and $\ln x$ to simplify expressions.
- Hw to compute complicated derivatives at given values.
- How to compute a Definite Integral involving Exponentials and Logs using FTC Part II. Know how to simplify combinations of exponential and log values. Review what we called the k-rule, it is for Antiderivatives only. Remember, you have two main tools: you can simplify the integrand using algebra, and/or you can do a substitution. The idea is to reduce the integrand to something you actually know the antiderivative of. Practice a combined technique of speed and accuracy.
- How to solve Initial Valued Differential Equations for a specific Antiderivative. The integral may involve u-substitution. Use the given value to solve for $+C$.
- How to compute integrals with u-substitution for **both** Indefinite (which gains $+C$) and Definite Integrals (which requires a **change of the limits** using the choice of u).
- How to compute derivatives of a Logarithm holding a large and complex input chunk. Use Log Algebra to simplify into smaller/simpler pieces before Differentiating.
- How to use *Logarithmic Differentiation*. Use this when the function has a variable in the base *and* the power, like x^x or $(\sin x)^x$. Or simpler, for other exponentials like $f(x) = 3^x$. Pratice proving $\dfrac{d}{dx}a^x = a^x \ln a$ where $a > 0$ is constant.
- How to solve equations with Exponentials or Logs, like $e^{6-5x} = 7$ or $\ln(4x - 7) = 9$.
- How to compute Absolute Max or Min values for Exponential and Log functions.
- How to compute Tangent Line Equations or values where Horizontal Tangents exist.
- How to use Curve Sketching techniques to sketch a new function involving e^x or $\ln x$.
- How to **Prove** $\dfrac{d}{dx}\ln x = \dfrac{1}{x}$
- How to compute Area bounded between curves. Make sure to sketch the graphs and shade the bounded regions. Make sure to solve for what x-values the curves intersect, if they are not given in the problem.
- How to distinguish between the need to use a Power Rule $\displaystyle\int \frac{1}{x^3}\,dx = \frac{x^{-2}}{-2} + C \ldots$ versus a new Log antiderivative $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$.