Math 106, Spring 2025

Homework #8

Due **Wednesday, February 26th** in Gradescope by 11:59 pm ET

Goal: Warm-Up Algebra for future Area Computations

FIRST: Read through and understand the following Examples.

Function Evaluation For these problems, i and n represent some constants. Evaluate the specific function value here in terms of i and n. Simplify the algebra by combining similar terms.

Ex: Consider $f(x) = x^2 - 3x - 7$. Compute

$$f\left(-1+\frac{3i}{n}\right) = \left(-1+\frac{3i}{n}\right)^2 - 3\left(-1+\frac{3i}{n}\right) - 7$$

$$= 1 - \frac{6i}{n} + \frac{9i^2}{n^2} + 3 - \frac{9i}{n} - 7 = \boxed{\frac{9i^2}{n^2} - \frac{15i}{n} - 3}$$

Ex: Consider $f(x) = x^2 - 5x + 2$. Compute

$$f\left(-2+\frac{6i}{n}\right) = \left(-2+\frac{6i}{n}\right)^2 - 5\left(-2+\frac{6i}{n}\right) + 2$$

$$= 4 - \frac{12i}{n} - \frac{12i}{n} + \frac{36i^2}{n^2} + 10 - \frac{30i}{n} + 2 = \boxed{\frac{36i^2}{n^2} - \frac{54i}{n} + 16}$$

Limit Evaluations Practice evaluating limits arising in future problems. Helpful algebra ...

Ex: $\lim_{n\to\infty} \frac{n+1}{n} \overset{\text{split}}{=} \lim_{n\to\infty} \frac{n}{n} + \frac{1}{n} = \lim_{n\to\infty} 1 + \frac{1}{n} = \lim_{n\to\infty} 1 + \frac{1}{n} \overset{\frac{1}{\infty}\to 0}{=} 1 + 0 = \boxed{1}$

Ex:

$$\lim_{n\to\infty} \frac{n(n+1)}{n^2} = \lim_{n\to\infty} \frac{n(n+1)}{n\cdot n} \overset{\text{partner}}{=} \lim_{n\to\infty} \left(\frac{n}{n}\right)\left(\frac{n+1}{n}\right)$$

$$\overset{\text{split}}{=} \lim_{n\to\infty} (1)\left(\frac{n}{n} + \frac{1}{n}\right) = \lim_{n\to\infty} (1)\left(1 + \frac{1}{n}\right) = \lim_{n\to\infty} 1 + \frac{1}{n} \overset{\frac{1}{\infty}\to 0}{=} \boxed{1}$$

Next, Complete the following Homework problems.

For problems 1-6, i and n are some constants. Simplify, combine similar variables.

1. Consider $f(x) = 6x + 5$. Compute $f\left(\frac{4i}{n}\right)$

2. Consider $f(x) = 6x + 5$. Compute $f\left(-2 + \frac{3i}{n}\right)$

3. Consider $f(x) = x^2 - 6x - 7$. Compute $f\left(\frac{4i}{n}\right)$

4. Consider $f(x) = x^2 - 6x - 7$. Compute $f\left(5 + \frac{2i}{n}\right)$

5. Consider $f(x) = x^2 - 5x - 4$. Compute $f\left(\frac{3i}{n}\right)$

6. Consider $f(x) = x^2 - 5x - 4$. Compute $f\left(-4 + \frac{5i}{n}\right)$

For problems 7-18, Evaluate each of the Limits. You may need to use algebra to decompose the Limit into simpler pieces. Also use *arrows* to show the size arguments, either growing large towards ∞ and/or small towards 0.

7. $\lim_{n\to\infty} 8$

8. $\lim_{n\to\infty} \frac{1}{n}$

9. $\lim_{n\to\infty} \frac{6}{n}$

10. $\lim_{n\to\infty} \frac{n+4}{n}$

11. $\lim_{n\to\infty} \frac{2n+1}{n}$

12. $\lim_{n\to\infty} \frac{n(n+4)}{n^2}$

13. $\lim_{n\to\infty} \frac{n(n+1)(4n+1)}{n^3}$

14. $\lim_{n\to\infty} \frac{6}{n^2}\left(\frac{n(n+1)}{2}\right)$

15. $\lim_{n\to\infty} \frac{9}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$

16. $\lim_{n\to\infty} \frac{15}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$

17. $\lim_{n\to\infty} \frac{36}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{100}{n^2}\left(\frac{n(n+1)}{2}\right) + \left(\frac{7}{n}\right)n$

18. $\lim_{n\to\infty} \frac{24}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{42}{n^2}\left(\frac{n(n+1)}{2}\right) + \left(\frac{5}{n}\right)n$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm **7:30–9:00 pm TA Andrew, SMUDD 207**

Tuesday: 1:00–4:00 pm

Wednesday: 1:00-3:00 pm

Thursday: none for Professor

8:00–9:30 pm TA Andrew, SMUDD 208A

Friday: 12:00–2:00 pm

- Present Final Drafts only please
- Justify all details, and show all steps