Math 106, Spring 2025

**Homework #3**

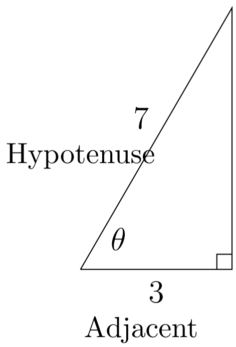
Due **Wednesday, February 5th** in Gradescope by 11:59 pm ET

**Goal:** More Trigonometry, Angles & Trigonometric Derivatives (including the Chain Rule).

**FIRST:** Read through and understand the following Examples.

Ex: Consider an angle $\theta$ where $0 \le \theta < \frac{\pi}{2}$. Suppose that $\cos\theta = \frac{3}{7}$. Find the value for both $\sin\theta$ and $\tan\theta$.

Here, $\cos\theta = \frac{3}{7} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ and we can set up the related triangle using the given Trig ratio




? $\sqrt{7^2 - 3^2} = \sqrt{49 - 9} = \sqrt{40}$

Opposite

As a result, we can read the sine and tangent off this reference triangle.

$$\sin\theta = \boxed{\frac{\sqrt{40}}{7}} = \frac{\text{Opposite}}{\text{Hypotenuse}} \quad \text{and} \quad \tan\theta = \boxed{\frac{\sqrt{40}}{3}} = \frac{\text{Opposite}}{\text{Adjacent}}$$

Ex: Consider $F(x) = \tan(2x) - \sin(3x)$. Compute $F'\left(\frac{\pi}{6}\right)$.

First, compute the derivative

$F'(x) = \sec^2(2x) \cdot 2 - \cos(3x) \cdot 3 = 2\sec^2(2x) - 3\cos(3x)$

Next, evaluate the derivative at the specific value $\frac{\pi}{6}$.

$$F'\left(\frac{\pi}{6}\right) = 2\sec^2\left(2\left(\frac{\pi}{6}\right)\right) - 3\cos\left(3\left(\frac{\pi}{6}\right)\right) = 2\sec^2\left(\frac{\pi}{3}\right) - 3\cos\left(\frac{\pi}{2}\right)$$

$$= \frac{2}{\cos^2\left(\frac{\pi}{3}\right)} - \cancelto{0}{3\cos\left(\frac{\pi}{2}\right)} = \frac{2}{\left(\cancelto{\frac{1}{2}}{\cos\left(\frac{\pi}{3}\right)}\right)^2} - 0 = \frac{2}{\left(\frac{1}{2}\right)^2} = \frac{2}{\frac{1}{4}} = 2 \cdot \frac{4}{1} = \boxed{8}$$

**Next, Complete the following Homework problems.**

For #1 – 2, evaluate the following Trig expressions, keeping $0 \leq \theta < \frac{\pi}{2}$

1. If $\sin\theta = \frac{1}{2}$, find $\cos\theta$ 2. If $\cos\theta = \frac{2}{5}$, find $\tan\theta$

For #3 – 4, use the facts $\boxed{\frac{d}{dx}\sin x = \cos x}$ and $\boxed{\frac{d}{dx}\cos x = -\sin x}$ to prove that

3. $\boxed{\frac{d}{dx}\tan x = \sec^2 x}$ and 4. $\boxed{\frac{d}{dx}\sec x = \sec x \tan x}$ **Memorize.**

For #5 – 6, solve for angle(s) $\theta$ in Radians keeping $0 \leq \theta < 2\pi$.

5. $\sin\theta = -\frac{1}{2}$ 6. $\sin\theta = -\frac{\sqrt{3}}{2}$

For #7 – 8, compute the following values. Justify. Show work on the Unit Circle/Trig Triangles.

7. $\cos\frac{4\pi}{3}$ 8. $\sin\frac{4\pi}{3}$

For #9 – 17, compute the Derivative for each of the following functions. Do **Not** simplify.

9. $y = \sin(x^2 - 5x + 8)$ 10. $f(x) = \sin^2 x$ 11. $y = \cos^6(3x)$

12. $y = \cos\sqrt{x}$ 13. $y = \sqrt{\cos x}$ 14. $f(x) = \dfrac{\cos(3x)}{\sin(4x)}$

15. $y = \tan\left(\dfrac{1}{x}\right)$ 16. $f(x) = \dfrac{1}{\tan x}$ 17. $y = \left(\dfrac{\cos x}{x^2 - \sin x}\right)^8$

18. Let $G(x) = \sin(2x) - \cos(3x)$. Compute $G'\left(\frac{\pi}{6}\right)$. Simplify your answer completely.

# REGULAR OFFICE HOURS

# Monday: 12:00–3:00 pm

# Tuesday: 1:00–4:00 pm

**TBA TA Andrew**

# Wednesday: 1:00-3:00 pm

# Thursday: none for Professor

**TBA TA Andrew**

# Friday: 12:00–2:00 pm

- We've finished a solid review of Trigonometry, and derivatives from Math 105. Aim to make clearer and neater solutions this week.

- Attend Office Hours regularly, both with Professor Benedetto and Math Fellow Andrew.