Math 106, Spring 2025

Homework #1

Due **Wednesday, January 29th** in Gradescope by 11:59 pm ET

Watch *Differentiation Review* Video

Goal: Review Derivatives, Tangent Lines & Differentiation Rules.

FIRST: Read through and understand the following Examples. Compute each Derivative.

Ex: $\dfrac{d}{dx}\left(\dfrac{5}{x^7}-\dfrac{4}{x^2}+x^7\right) \overset{\text{prep}}{=} \dfrac{d}{dx}\left(5x^{-7}-4x^{-2}+x^7\right) = -35x^{-8}+8x^{-3}+7x^6 = \boxed{-\dfrac{35}{x^8}+\dfrac{8}{x^3}+7x^6}$

Ex: Consider $f(x) = x^3 - 5x^5 - \dfrac{3}{5} + \dfrac{5}{x^3} + x^{\frac{3}{5}} + \dfrac{1}{x^{\frac{5}{3}}} \overset{\text{prep}}{=} x^3 - 5x^5 - \dfrac{3}{5} + 5x^{-3} + x^{\frac{3}{5}} + x^{-\frac{5}{3}}$

Differentiate $f'(x) = 3x^2-25x^4-0-15x^{-4}+\dfrac{3}{5}x^{-\frac{2}{5}}-\dfrac{5}{3}x^{-\frac{8}{3}} = \boxed{3x^2 - 25x^4 - \dfrac{15}{x^4} + \dfrac{3}{5x^{\frac{2}{5}}} - \dfrac{5}{3x^{\frac{8}{3}}}}$

Ex: $g(x) = \sqrt{x^7 - \dfrac{9}{x^2}} \overset{\text{prep}}{=} \sqrt{x^7 - 9x^{-2}}$

Differentiate $g'(x) = \dfrac{1}{2\sqrt{x^7 - \dfrac{9}{x^2}}} \cdot \left(7x^6 + 18x^{-3}\right) = \boxed{\dfrac{7x^6 + \dfrac{18}{x^3}}{2\sqrt{x^7 - \dfrac{9}{x^2}}}}$ Chain Rule

Ex: $H(x) = \dfrac{7-3x}{2x+5}$

$H'(x) = \dfrac{(2x+5)(-3)-(7-3x)\cdot 2}{(2x+5)^2} = \dfrac{\cancel{-6x} - 15 - 14 + \cancel{6x}}{(2x+5)^2} = \boxed{\dfrac{-29}{(2x+5)^2}}$ Quotient Rule

Ex: $W(x) = (8-5x^2)\left(\sqrt{x} - \dfrac{6}{x^2}\right) \overset{\text{FOIL}}{=} 8\sqrt{x} - 48x^{-2} - 5x^{\frac{5}{2}} + 30$ Simplify first

THEN Differentiate second $W'(x) = \dfrac{8}{2\sqrt{x}} + 96x^{-3} - \dfrac{25}{2}x^{\frac{3}{2}} + 0 = \boxed{\dfrac{4}{\sqrt{x}} + \dfrac{96}{x^3} - \dfrac{25}{2}x^{\frac{3}{2}}}$

OR Differentiate first using Product Rule **THEN** second simplify using algebra

$$\begin{aligned} W'(x) &= (8-5x^2)\left(\frac{1}{2\sqrt{x}} + 12x^{-3}\right) + \left(\sqrt{x} - \frac{6}{x^2}\right)(-10x) \\ &= \frac{4}{\sqrt{x}} + 96x^{-3} - \frac{5}{2}x^{\frac{3}{2}} - \cancel{60x^{-1}} - 10x^{\frac{3}{2}} + \cancel{60x^{-1}} = \boxed{\frac{4}{\sqrt{x}} + \frac{96}{x^3} - \frac{25}{2}x^{\frac{3}{2}}} \; \textit{Match!} \end{aligned}$$

Next, Complete the following Homework problems.

Plenty of help in Office Hours!

For each HW, **match** your final answers to the posted Answer Key.

For problems $\#1-11$, Compute the derivative of the following functions.

1. $f(x)=\pi^2$
2. $f(x)=\frac{3}{4}x^8$
3. $f(x)=x^3-4x+6$
4. $f(x)=x^7-6x^5-9$
5. $f(x)=\frac{1}{x^3}$
6. $f(x)=-\frac{12}{x^5}$
7. $f(x)=5x^2-\sqrt{x}+x^{\frac{5}{7}}$
8. $y=\frac{3}{x}-\frac{2}{x^2}$
9. $y=\sqrt{8x^5-x^9}$
10. $y=\left(\sqrt{x}+\frac{1}{x}\right)^9$
11. $f(x)=\frac{6}{7}x-x^{\frac{6}{7}}+\frac{1}{7x^6}-\frac{1}{6}$

12. Compute the derivative of $f(x)=\sqrt{x}$ **two** different ways:

- First use the **Limit Definition of the Derivative** $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
- Second use the Power Rule.

IMPORTANT: Now **MEMORIZE** $\boxed{\frac{d}{dx}\sqrt{x}=\frac{1}{2\sqrt{x}}}$

For problems $\#13-14$ compute the derivative of each of the following functions in **two** ways:

- FIRST simplify the function algebra and THEN use the Sum and Power Differentiation Rules.
- FIRST use the Product or Quotient Differentiation Rules and THEN simplify and show that it matches your answer from above.

13. $f(x)=(x^5+5)\left(\frac{1}{x}-3x^2\right)$
14. $f(x)=\dfrac{3x^2-\frac{1}{x^8}}{x^7}$

15. Compute the equation of the Tangent Line to $f(x)=\frac{2}{3}x^{\frac{3}{2}}+2\sqrt{x}$ at the point where $x=4$.

Recall $4^{\frac{3}{2}}=\left(4^{\frac{1}{2}}\right)^3=\left(\sqrt{4}\right)^3$

16. Consider $f(x)=\frac{4x+3}{1-5x}$. Compute $f'(x)$. Simplify.

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

Tuesday: 1:00–4:00 pm

Wednesday: 1:00-3:00 pm

Friday: 12:00–2:00 pm

Math Fellow evening TA Help Hours TBD soon

• Please take the chance to stop by my office hours (unannounced) to meet me and ask any questions you have. If you can't make these hours, go ahead and make an appointment at a different time. I will try to accomodate everyone. Just e-mail me.

• Please turn in your homework by the deadline. Gradescope will lock you out at 11:59 pm.

• Please **TAG** your HW solution numbers in Gradescope. Ask for help if needed

• Write a **final** draft neatly in either pen or pencil. No mess. No Scratch-outs. Make sure that your scan is clear for all problems.

• You are responsible for writing up your own solutions, in your own words. Please read the Statement of Intellectual Responsibility from our class syllabus. **No** online solution sources. I will give zero credit for all work copied from any other source. I will also report you to the Dean of Conduct/Community Standards. You will also risk immediate Failure in the course.

• **NO LATE HOMEWORK!** unless illness or emergency occurs.