

Exam 2 Spring 2024

$$1(a) \int_{-1}^2 3 - 4x - x^2 dx = 3x - \frac{4x^2}{2} - \frac{x^3}{3} \bigg|_{-1}^2 = 3(2) - 2(2)^2 - \frac{2^3}{3} - \left(3(-1) - 2(-1)^2 - \frac{(-1)^3}{3} \right)$$

$$= 6 - 8 - \frac{8}{3} + 3 + 2 - \frac{1}{3} = \cancel{8} - \cancel{8} + 3 - \frac{9}{3} = 3 - 3 = 0 \text{ Match!}$$

$$1(b) \int_{-1}^2 3 - 4x - x^2 dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \cdot \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(-1 + \frac{3i}{n}\right) \cdot \frac{3}{n}$$

$f(x) = 3 - 4x - x^2$
 $a = -1 \quad b = 2$
 $\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$
 $x_i = a + i\Delta x = -1 + \frac{3i}{n}$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(3 - 4\left(-1 + \frac{3i}{n}\right) - \left(-1 + \frac{3i}{n}\right)^2 \right) \frac{3}{n}$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n 3 + 4 - \frac{12i}{n} - 1 + \frac{6i}{n} - \frac{9i^2}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n 6 - \frac{6i}{n} - \frac{9i^2}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n 6 - \frac{3}{n} \sum_{i=1}^n \frac{6i}{n} - \frac{3}{n} \sum_{i=1}^n \frac{9i^2}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{18}{n} \sum_{i=1}^n 1 - \frac{18}{n^2} \sum_{i=1}^n i - \frac{27}{n^3} \sum_{i=1}^n i^2$$

$$= \lim_{n \rightarrow \infty} \frac{18}{n} \cdot n - \frac{18}{n^2} \left(\frac{n(n+1)}{2} \right) - \frac{27}{n^3} \left(\frac{n(n+1)(2n+1)}{6} \right)$$

$$= \lim_{n \rightarrow \infty} 18 - \frac{18}{2} \left(\frac{n}{n} \right) \left(\frac{n+1}{n} \right) - \frac{27}{6} \left(\frac{n}{n} \right) \left(\frac{n+1}{n} \right) \left(\frac{2n+1}{n} \right)$$

$$= \lim_{n \rightarrow \infty} 18 - 9 \left(1 \right) \left(1 + \frac{1}{n} \right) - \frac{27}{6} \left(1 \right) \left(1 + \frac{1}{n} \right) \left(2 + \frac{1}{n} \right)$$

$$= 18 - 9 - \frac{27}{3} \cdot 2 = 9 - 9 = 0 \text{ Match!}$$

$$2(a) \int \frac{5}{\sqrt{x}(3+\sqrt{x})^7} dx = 5 \cdot 2 \int \frac{1}{u^7} du = \cancel{10} \cdot \frac{u^{-6}}{\cancel{-6}_3} + C = -\frac{5}{3} \left(\frac{1}{(3+\sqrt{x})^6} \right) + C$$

$$u = 3 + \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}} dx$$

$$2du = \frac{1}{\sqrt{x}} dx$$

$$2(b) \int \frac{\sec^2 x}{(5+\tan x)^3} dx = \int \frac{1}{u^3} du = \frac{u^{-2}}{-2} + C = -\frac{1}{2(5+\tan x)^2} + C$$

$$u = 5 + \tan x$$

$$du = \sec^2 x dx$$

$$2(c) \int \frac{X}{(x-3)^9} dx = \int \frac{u+3}{u^9} du = \int \frac{u}{u^9} + \frac{3}{u^9} du = \int u^{-8} + 3u^{-9} du$$

$$u = x-3 \Rightarrow x = u+3$$

$$du = dx$$

$$= \frac{u^{-7}}{-7} + 3 \frac{u^{-8}}{-8} + C$$

$$= -\frac{1}{7(x-3)^7} - \frac{3}{8(x-3)^8} + C$$

$$3(a) \int_{\frac{\pi}{9}}^{\frac{\pi}{3}} \sin(3x) + \sqrt{3} \cos(6x) dx = \int_{\frac{\pi}{9}}^{\frac{\pi}{3}} \sin(3x) dx + \sqrt{3} \int_{\frac{\pi}{9}}^{\frac{\pi}{3}} \cos(6x) dx$$

$$u = 3x \\ du = 3 dx \\ \frac{1}{3} du = dx$$

$$= \frac{1}{3} \int_{\frac{\pi}{3}}^{\pi} \sin u du + \frac{\sqrt{3}}{6} \int_{\frac{2\pi}{3}}^{2\pi} \cos w dw$$

$$w = 6x \\ dw = 6 dx \\ \frac{1}{6} dw = dx$$

$$x = \frac{\pi}{9} \Rightarrow u = 3 \cdot \frac{\pi}{9} = \frac{\pi}{3}$$

$$x = \frac{\pi}{3} \Rightarrow u = 3 \cdot \frac{\pi}{3} = \pi$$

$$= -\frac{1}{3} \cos u \Big|_{\frac{\pi}{3}}^{\pi} + \frac{\sqrt{3}}{6} \sin w \Big|_{\frac{2\pi}{3}}^{2\pi}$$

$$x = \frac{\pi}{9} \Rightarrow u = 6 \cdot \frac{\pi}{9} = \frac{2\pi}{3}$$

$$x = \frac{\pi}{3} \Rightarrow u = 6 \cdot \frac{\pi}{3} = 2\pi$$

$$= -\frac{1}{3} \cos \pi - \left(-\frac{1}{3} \cos \frac{\pi}{3} \right) + \frac{\sqrt{3}}{6} \sin(2\pi) - \frac{\sqrt{3}}{6} \sin\left(\frac{2\pi}{3}\right)$$

$$= \frac{1}{3} + \frac{1}{6} + 0 - \frac{3}{12} = \frac{4}{12} + \frac{2}{12} - \frac{3}{12} = \frac{3}{12} = \frac{1}{4} \text{ Match!}$$

$$3(b) \int_{-\pi}^{3\pi} \cos\left(\frac{x}{2}\right) dx = 2 \int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos u du = 2 \sin u \Big|_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} = 2 \cdot \left(\sin \frac{3\pi}{2} - \sin\left(-\frac{\pi}{2}\right) \right)$$

$$= 2 \cdot \left(\cancel{-1} + \cancel{1} \right) = 0 \text{ Match!}$$

$$\stackrel{or}{=} = -2 + 2 = 0$$

$$\begin{aligned} u &= \frac{x}{2} = \frac{1}{2} \cdot x \\ du &= \frac{1}{2} dx \\ 2du &= dx \end{aligned}$$

$$\begin{aligned} x = -\pi &\Rightarrow u = -\frac{\pi}{2} \\ x = 3\pi &\Rightarrow u = \frac{3\pi}{2} \end{aligned}$$

$$3(c) \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{\cos^3 x} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{(\cos x)^3} dx = - \int_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}} \frac{1}{u^3} du = - \frac{u^{-2}}{\cancel{-2}} \Big|_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}} = \frac{1}{2u^2} \Big|_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}}$$

$$\begin{aligned} u &= \cos x \\ du &= -\sin x dx \\ -du &= \sin x dx \end{aligned}$$

$$\begin{aligned} x = \frac{\pi}{6} &\Rightarrow u = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \\ x = \frac{\pi}{3} &\Rightarrow u = \cos \frac{\pi}{3} = \frac{1}{2} \end{aligned}$$

$$= \frac{1}{2} \left(\frac{1}{\left(\frac{1}{2}\right)^2} - \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} \right) = \frac{1}{2} \left(4 - \frac{4}{3} \right)$$

$$= \frac{1}{2} \left(\frac{12}{3} - \frac{4}{3} \right) = \frac{1}{\cancel{2}} \left(\frac{8}{3} \right) = \frac{4}{3} \text{ Match!}$$

$$4. \int_{-1}^3 |x-2| + 1 dx = \int_{-1}^2 \cancel{x-2} + 1 dx + \int_2^3 \cancel{x-2} + 1 dx$$

Cases

$$|x-2| = \begin{cases} x-2 & \text{if } x-2 \geq 0 \\ -(x-2) & \text{if } x-2 < 0 \end{cases}$$

$$= \begin{cases} x-2 & \text{if } x \geq 2 \\ -x+2 & \text{if } x < 2 \end{cases}$$

$$= \int_{-1}^2 -x+3 dx + \int_2^3 x-1 dx$$

$$= \left. -\frac{x^2}{2} + 3x \right|_{-1}^2 + \left. \frac{x^2}{2} - x \right|_2^3$$

$$= -\frac{4}{2} + 6 - \left(-\frac{1}{2} - 3 \right) + \frac{9}{2} - 3 - \left(\frac{4}{2} - 2 \right)$$

$$= -2 + 6 + \frac{1}{2} + 3 + \frac{9}{2} - 3$$

$$= 4 + \frac{10}{2} = 4 + 5 = 9 \text{ Match!}$$

$$5. f'(x) = \frac{1}{x^3 \sqrt{3 + \frac{6}{x^2}}} \quad \text{and} \quad f(1) = -\frac{5}{2}$$

$$f(x) = \int f'(x) dx = \int \frac{1}{x^3 \sqrt{3 + \frac{6}{x^2}}} dx = -\frac{1}{12} \int \frac{1}{\sqrt{u}} du = -\frac{1}{12} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$\begin{aligned} u &= 3 + \frac{6}{x^2} \\ du &= -\frac{12}{x^3} dx \\ -\frac{1}{12} du &= \frac{1}{x^3} dx \end{aligned}$$

$$= -\frac{1}{12} \cdot 2\sqrt{u} + C = -\frac{1}{6} \sqrt{3 + \frac{6}{x^2}} + C$$

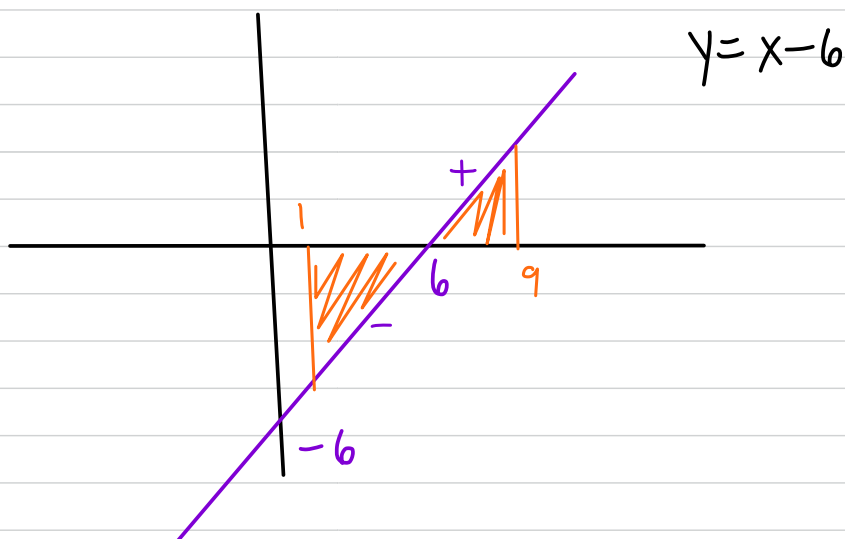
Use Initial Value

$$f(1) = -\frac{1}{6} \sqrt{3+6} + C = -\frac{5}{2} \quad \text{set}$$

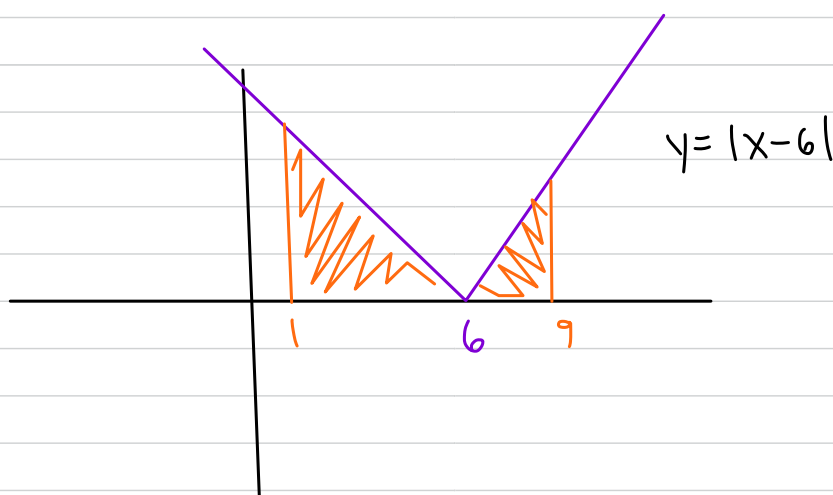
$$-\frac{1}{2} + C = -\frac{5}{2} \Rightarrow C = -\frac{4}{2} = -2$$

$$\text{Finally, } f(x) = -\frac{1}{6} \sqrt{3 + \frac{6}{x^2}} - 2$$

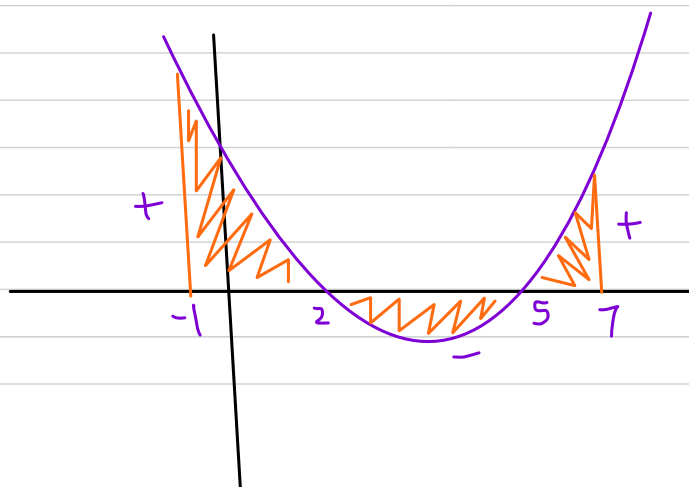
$$6(a) \int_1^9 x-6 dx = \text{Net Area}$$



$$6(b) \int_1^9 |x-6| dx$$



$$6(c) \int_{-1}^7 x^2 - 7x + 10 dx = \text{Net Area}$$



$$\text{Factor} \quad x^2 - 7x + 10 = (x-2)(x-5) = 0$$

$$x=2 \quad x=5$$

Parabola "zeroes"