

Exam 1 Spring 2025 Answer Key

1(a) $f(x) = \tan\left(\frac{\pi}{6}\right) + \tan\left(\frac{6}{x}\right) \stackrel{\text{prep}}{=} \tan\left(\frac{\pi}{6}\right) + \tan\left(6x^{-1}\right)$

$$f'(x) = 0 + \sec^2\left(\frac{6}{x}\right) \cdot (-6x^{-2})$$

Do Not Need to Simplify

1(b) $f(x) = \cos(\sin(\sec x))$

$$f'(x) = -\sin(\sin(\sec x)) \cdot \cos(\sec x) \cdot \sec x \tan x$$

1(c) $f(x) = \sin^6\left(\frac{5}{x^4}\right) \stackrel{\text{prep}}{=} \left(\sin\left(\frac{5}{x^4}\right)\right)^6$

prep $5x^{-4} \xrightarrow{\frac{d}{dx}} -20x^{-5}$

$$f'(x) = 6 \left(\sin\left(\frac{5}{x^4}\right)\right)^5 \cdot \cos\left(\frac{5}{x^4}\right) \cdot (-20x^{-5})$$

1(d) $f(x) = 5\sin^2 x + 5\cos^2 x \stackrel{\text{prep}}{=} 5(\sin^2 x + \cos^2 x) \stackrel{!}{=} 5 \text{ constant}$

$$f'(x) = 0$$

or

$$f(x) = 5(\sin x)^2 + 5(\cos x)^2$$

$$f'(x) = 10(\sin x)^1 \cdot \cos x + 10(\cos x)^1 \cdot (-\sin x)$$

$$= 10 \sin x \cos x - 10 \sin x \cos x = 0$$

1(e) $f(x) = \frac{6}{\sqrt{\cos \sqrt{x}}} \stackrel{\text{prep}}{=} 6(\cos \sqrt{x})^{-1/2}$

$$f'(x) = -3(\cos \sqrt{x})^{-3/2} \cdot (-\sin \sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

2(a) $f(x) = \cos(7x) + \cos(6x) + \sin(3x) + \sin(4x)$

$$f'(x) = -7\sin(7x) - 6\sin(6x) + 3\cos(3x) + 4\cos(4x)$$

$$f'\left(\frac{\pi}{6}\right) = -7\sin\left(\frac{7\pi}{6}\right) - 6\sin\left(\frac{6\pi}{6}\right) + 3\cos\left(\frac{3\pi}{6}\right) + 4\cos\left(\frac{4\pi}{6}\right)$$

$$= -7\sin\left(\frac{7\pi}{6}\right) - 6\sin(\pi) + 3\cos\left(\frac{\pi}{2}\right) + 4\cos\left(\frac{2\pi}{3}\right)$$

See 4. above

See 5. above

$$= -\frac{7}{2} + 0 + 0 - 2 = \frac{7}{2} - 2 = \frac{7}{2} - \frac{4}{2} = \frac{3}{2} \text{ Match}$$

$$2(b) \quad H(x) = \cos^2(2x) + \sin(6x) + 2\sin x$$

$$= (\cos(2x))^2 + \sin(6x) + 2\sin x$$

$$H'(x) = 2\cos(2x) \cdot (-\sin(2x)) \cdot 2 + 6\cos(6x) + 2\cos x$$

$$H'\left(\frac{\pi}{6}\right) = -4\cos\left(\frac{\pi}{3}\right) \cdot \sin\left(\frac{\pi}{3}\right) + 6\cos\left(\frac{\pi}{2}\right) + 2\cos\left(\frac{\pi}{6}\right)$$

$$= -4\cos\left(\frac{\pi}{3}\right) \cdot \sin\left(\frac{\pi}{3}\right) + 6\cos\left(\frac{\pi}{2}\right) + 2\cos\left(\frac{\pi}{6}\right)$$

$$= -4\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + 6(-1) + 2 \cdot \frac{\sqrt{3}}{2}$$

$$= -\sqrt{3} - 6 + \sqrt{3} = -6 \quad \text{Match!}$$

$$3(a) \quad \int \frac{3}{7}x + x^{\frac{3}{7}} + \frac{7}{3}x^3 - \frac{1}{x^{\frac{3}{7}}} + \frac{1}{7} + \frac{1}{3x^{\frac{7}{3}}} - \frac{1}{7x^3} - \frac{3}{x^7} dx$$

$$\stackrel{\text{prep}}{=} \int \frac{3}{7}x + x^{\frac{3}{7}} + \frac{7}{3}x^3 - x^{-\frac{3}{7}} + \frac{1}{7} + \frac{1}{3}x^{-\frac{7}{3}} - \frac{1}{7}x^{-3} - 3x^{-7} dx$$

Don't Move Constants

$$= \frac{3}{7} \cdot \frac{x^2}{2} + \frac{x^{\frac{10}{7}}}{\frac{10}{7}} + \frac{7}{3} \cdot \frac{x^4}{4} - \frac{x^{\frac{4}{7}}}{\frac{4}{7}} + \frac{1}{7}x + \frac{1}{3} \left(\frac{x^{-\frac{4}{3}}}{-\frac{4}{3}} \right) - \frac{1}{7} \left(\frac{x^{-2}}{-2} \right) - \frac{3x^{-6}}{-6} + C$$

$$= \frac{3x^2}{14} + \frac{7}{10}x^{\frac{10}{7}} + \frac{7}{12}x^4 - \frac{7}{4}x^{\frac{4}{7}} + \frac{1}{7}x - \frac{3}{4} \cdot \frac{1}{3}x^{-\frac{4}{3}} + \frac{1}{14}x^{-2} + \frac{1}{2}x^{-6} + C$$

$$3(b) \quad \int \left(x^3 + \frac{1}{x^3}\right) \left(x - \frac{1}{x}\right) dx \stackrel{\text{FOIL}}{=} \int x^4 - x^2 + x^{-2} - x^{-4} dx$$

$$= \frac{x^5}{5} - \frac{x^3}{3} + \frac{x^{-1}}{-1} - \frac{x^{-3}}{-3} + C$$

$$= \frac{x^5}{5} - \frac{x^3}{3} - \frac{1}{x} + \frac{1}{3x^3} + C$$

$$3(c) \quad \int \frac{x^7 - 4x^3 - \frac{8}{x} + \sqrt{x} + 5 - x^3 \sec^2 x}{x^3} dx \stackrel{\text{split}}{=} \int \frac{x^7}{x^3} - \frac{4x^3}{x^3} - \frac{8x^{-1}}{x^3} + \frac{x^{\frac{1}{2}}}{x^3} + \frac{5}{x^3} - \frac{x^3 \sec^2 x}{x^3} dx$$

$$= \int x^4 - 4 - 8x^{-4} + x^{-\frac{5}{2}} + 5x^{-3} - \sec^2 x dx$$

$$= \frac{x^5}{5} - 4x - \frac{8x^{-3}}{-3} + \frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} + \frac{5x^{-2}}{-2} - \tan x + C$$

$$= \frac{x^5}{5} - 4x + \frac{8}{3x^3} - \frac{2}{3x^{\frac{3}{2}}} - \frac{5}{2x^2} - \tan x + C$$

4. Prove $\frac{d}{dx} (\tan x) = \sec^2 x$

$$\frac{d}{dx} (\tan x) = \frac{d}{dx} \left(\frac{\sin x}{\cos x} \right) = \frac{\cos x \cdot (\cos x) - (\sin x) \cdot (-\sin x)}{(\cos x)^2} \quad \text{Quotient Rule}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \left(\frac{1}{\cos x} \right)^2 = \sec^2 x \quad \text{Trig Identity} \quad \text{Match}$$

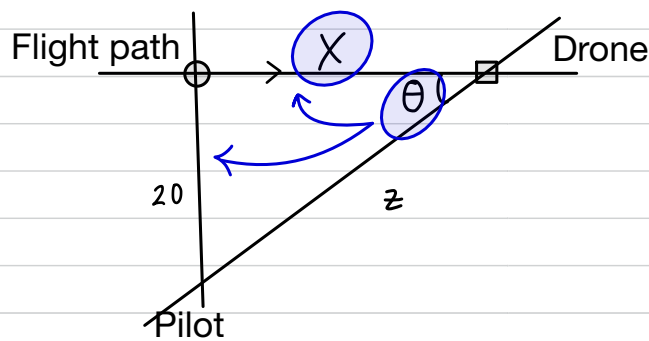
OR

split

$$= \frac{\cancel{\cos^2 x} + \sin^2 x}{\cancel{\cos^2 x}} = 1 + \tan^2 x = \sec^2 x$$

5. Diagram

→ given



Variables

Let x = Distance the drone has travelled horizontally

z = Distance between drone and pilot

θ = Angle at the drone corner

Given $\frac{dx}{dt} = 10$ feet/sec

Find $\frac{d\theta}{dt} = ?$ when $z = 40$

Equation

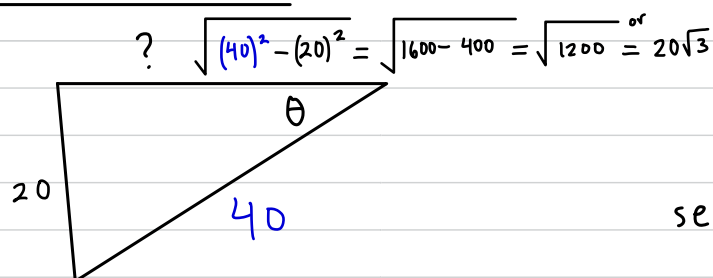
$$\tan \theta = \frac{20}{x}$$

Differentiate

$$\frac{d}{dt} (\tan \theta) = \frac{d}{dt} (20x^{-1})$$

$$(\sec \theta)^2 \cdot \frac{d\theta}{dt} = -20x^{-2} \cdot \frac{dx}{dt}$$

Extra Solvable Information



$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{A}{H}} = \frac{H}{A} = \frac{40}{\sqrt{1200}}$$

Substitute

$$\left(\frac{40}{\sqrt{1200}} \right)^2 \cdot \frac{d\theta}{dt} = -\frac{20}{(\sqrt{1200})^2} \cdot (10)$$

$$\frac{(40)^2}{(\sqrt{1200})^2}$$

Solve

$$\frac{d\theta}{dt} = -\frac{20}{1200} (10) \cdot \frac{1200}{1600}$$

Minus makes sense since Angle shrinking as drone flies

$$= -\frac{200}{1600} = -\frac{1}{8} \text{ Radians/Sec.}$$

Answer

The Angle at the drone corner is decreasing $\frac{1}{8}$ Radians every second at that Moment

6. $G'(x) = \frac{18}{x^2} - \frac{1}{\sqrt{x}} + 3$

$$G(x) = \int G'(x) dx = \int \frac{18}{x^2} - \frac{1}{\sqrt{x}} + 3 dx$$

$18x^{-2} - x^{-1/2}$

$$= \frac{18x^{-1}}{-1} - \frac{x^{1/2}}{1/2} + 3x + C$$

$$= -\frac{18}{x} - 2\sqrt{x} + 3x + C$$

$$G(9) = -\frac{18}{9} - 2\sqrt{9} + 27 + C \stackrel{\text{set}}{=} 7$$

$$-2 - 6 + 27 + C = 7$$

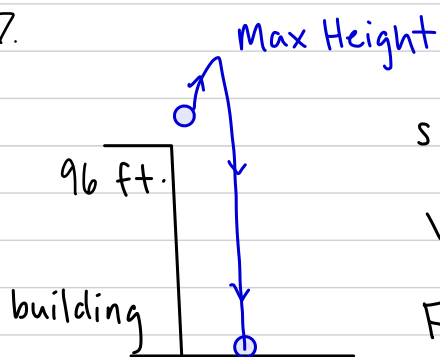
-8

$$19 + C = 7$$

$$\hookrightarrow C = -12$$

Finally, $G(x) = -\frac{18}{x} - 2\sqrt{x} + 3x - 12$

7.



$$s(0) = 96 \text{ ft.}$$

$$v(0) = 80 \text{ ft/sec}$$

$$\text{Find } t_{\max} = ?$$

$$\text{Find Max Height } \hookrightarrow s(t_{\max}) = ?$$

$$\text{Find } t_{\text{impact}} = ?$$

$$\text{Find } v(t_{\text{impact}}) = ?$$

$$a(t) = -32$$

$$v(t) = -32t + \cancel{v_0}^{80}$$

$$= -32t + 80$$

$$s(t) = -16t^2 + \cancel{v_0}^{80}t + \cancel{s_0}^{96}$$

$$= -16t^2 + 80t + 96$$

(a) Max Height is when $v(t)=0$

$$v(t) = -32t + 80 \stackrel{\text{set}}{=} 0 \rightarrow \text{Solve } 32t = 80 \Rightarrow t_{\max} = \frac{80}{32} = \frac{10}{4} = \frac{5}{2} \text{ sec}$$

Max Height occurs at $t = \frac{5}{2}$ seconds

2.5

(b) Max Height occurs when $t_{\max} = \frac{5}{2}$ seconds

$$\Rightarrow s(t_{\max}) = s\left(\frac{5}{2}\right) = -16\left(\frac{5}{2}\right)^2 + \cancel{80}^{\cancel{40}}\left(\frac{5}{\cancel{2}}\right) + 96 \quad \text{Fractions will cancel}$$

$$= -\cancel{16}^{\cancel{4}}\left(\frac{25}{\cancel{4}}\right) + 200 + 96$$

$$= \underbrace{-100 + 200}_{100} + 96 = 196 \text{ feet}$$

The Maximum Height is 196 feet

(c) Strikes ground when $s(t)=0$

$$s(t) = -16t^2 + 80t + 96 = -16(t^2 - 5t - 6)$$

$$= -16(t-6)(t+1) \stackrel{\text{set}}{=} 0$$

$$\swarrow \quad \searrow$$
$$t-6=0 \quad \text{or} \quad t+1=0$$

$$t_{\text{impact}} = 6 \text{ Sec.}$$

$$\cancel{t = -1 \text{ sec}}$$

Hits ground after $t_{\text{impact}} = 6$ seconds

$$(d) \text{ Velocity at impact } v(t_{\text{impact}}) = v(6) = -32 \cdot 6 + 80 = -192 + 80 = -112 \text{ ft/sec}$$

Impact Velocity is -112 feet per second

negative makes sense. DOWN @ impact.