Math 106, Spring 2024

Worksheet 3, Tuesday, February 20th, 2024

1. Differentiate each of the following functions. **Do NOT** simplify your answers here.

(a) $f(x) = \sin\left(\cos^6\left(\dfrac{9}{x^8}\right)\right) \stackrel{\text{prep}}{=} \sin\left[\left(\cos\left(\dfrac{9}{x^8}\right)\right)^6\right]$

(b) $y = \tan\left(\dfrac{9}{\sin x}\right) \stackrel{\text{prep}}{=} \tan\left(9\left(\sin x\right)^{-1}\right)$ 　　(c) $g(t) = \dfrac{8 + \sec(7t^2)}{9 + \cos t}$

2. Consider the function $f(x) = \cos^2(2x) + \tan(2x) + \sin(6x) + \sqrt{3} \cdot x$. Compute $f'\left(\dfrac{\pi}{6}\right)$.

3. Sally is standing 10 meters from a railroad track as a train goes past. She is waiting for her friend Bob, who is on the train looking at Sally through the window. The train misses its stop. At the moment when the distance between Sally and Bob is 13 meters, Bob's head is rotating at a rate of 2 radians per second to keep her in sight. How fast is the train going at that moment?

4. Find $\sin\left(\dfrac{7\pi}{6}\right)$ and $\cos\left(\dfrac{2\pi}{3}\right)$. Justify using Unit Circle/Trig Triangles.

5. Consider the function $f(x) = \cos(7x) + \cos(6x) + \sin(3x) + \sin(4x)$. Compute $f'\left(\dfrac{\pi}{6}\right)$. Problem 4 will be helpful here.

6. Solve for angle(s) θ in Radians keeping $0 \le \theta < 2\pi$, where $\sin\theta = -\dfrac{1}{2}$.

7. Find a function f such that $f''(x) = 20x^3 + 12x^2 + 4$ and $f(0) = 8$ and $f(1) = 5$.

8. Find the Most General Antiderivative for each of the following:

(a) $\displaystyle\int \dfrac{7x^{\frac{2}{5}} + 8x^{-\frac{4}{3}} + \dfrac{1}{x}}{\sqrt{x}}\, dx$ 　　(b) $\displaystyle\int \left(\sqrt{3} + \dfrac{1}{x^3}\right)\left(x - \dfrac{1}{x^{\frac{2}{7}}}\right)\, dx$

9. Mark throws a baseball upward from the top of a bridge. The initial *speed* of the ball is 80 feet per second. It hits the ground with a *speed* of 112 feet per second. How tall is the bridge? Hint: Use $a(t) = -32$ feet per second squared as acceleration due to gravity.

Turn in your own solutions into Gradescope before 11:59 pm today, Tuesday Feb 20

Finish all problems through number 9