Math 106, Spring 2024

Worksheet 2, Tuesday, February 13th, 2024
Antiderivatives

Definition: Take F, f functions defined on an interval I and suppose that $F'(x) = f(x)$ on I. Then

- $F(x)$ is called **AN** Antiderivative of $f(x)$
- $F(x) + C$ is called **the Most General** Antiderivative of $f(x)$, where C is any constant Real Number.

We will use the notation $\int f(x)\,dx$ to denote the Most General Antiderivative. The symbols $\int$ and dx are partnered-together *instructions* to compute the Most General Antiderivative.

For example: $\int x^7\,dx = \frac{x^8}{8} + C$ where $+C$ represent all possible constants.
This means that $\frac{d}{dx}\left(\frac{x^8}{8} + C\right) = x^7$ which looks correct to us.

Note that $\frac{x^8}{8} + 3$ is **an** antiderivative of x^7. So is $\frac{x^8}{8} + 2022$ as well as $\frac{x^8}{8} - 5$ and $\frac{x^8}{8} + \sqrt{3}$.

Hint: if you ever want to know whether you found the correct antiderivative, take the derivative of your answer and check that you return to the original function.

1. Write a *general power rule* for $\int x^n\,dx$ where n is any real number with $n \neq -1$.
(We will learn the $n = -1$ case at the very end of this semester.)

Compute each of the following Most General Antiderivatives

2. $\int \sqrt{x}\,dx$ 3. $\int \frac{1}{x^9}\,dx$ 4. $\int \frac{1}{\sqrt{x}}\,dx$ 5. $\int \frac{1}{x^{\frac{3}{7}}}\,dx$

6. $\int \cos x\,dx$ 7. $\int \sin x\,dx$ 8. $\int \sec^2 x\,dx$ 9. $\int \sec x \tan x\,dx$

For $\#10 - 11$, consider $f(x) = \frac{3}{4}x + x^{\frac{3}{4}} - \frac{1}{x^{\frac{3}{4}}} + \frac{4}{3} + \frac{1}{x^{\frac{4}{3}}} + \frac{3}{4x^4} - \frac{4}{x^3}$ Hint: prep $\frac{1}{x^a} = x^{-a}$

10. Compute $f'(x) = \frac{d}{dx}\left(\frac{3}{4}x + x^{\frac{3}{4}} - \frac{1}{x^{\frac{3}{4}}} + \frac{4}{3} + \frac{1}{x^{\frac{4}{3}}} + \frac{3}{4x^4} - \frac{4}{x^3}\right)$

11. Compute $\int f(x)\,dx = \int \frac{3}{4}x + x^{\frac{3}{4}} - \frac{1}{x^{\frac{3}{4}}} + \frac{4}{3} + \frac{1}{x^{\frac{4}{3}}} + \frac{3}{4x^4} - \frac{4}{x^3}\,dx$

Find the Most General Antiderivative of the following functions. You may need to use some Algebra to *prep* the function pieces for each *Power Rule* and/or *Sum or Difference* Rule(s).

12. $\int x^3(1+x^2)\,dx$

13. $\int \frac{x+\sqrt{x}+7}{x^3}\,dx$

14. $\int x^2+x(1+x)^2\,dx$

15. $\int -3\cos x-\sec^2 x-7\sec x\tan x-\sin x\,dx$

16. Consider the curve $y=2x+\sin x$. Explain why the tangent lines of this curve are never horizontal.

Find the indicated functions $f(x)$ that also satisfies the given conditions:

17. $f'(x)=2+\sin x$ and $f\left(\frac{\pi}{2}\right)=3$.

18. $f'(x)=x^2+1$ and $f(1)=3$.

19. $f'(x)=x(2+\sqrt{x})$ and $f(4)=30$.

20. $f''(x)=\frac{1}{\sqrt{x}}+3x^2$ and $f'(1)=2,\quad f(1)=0$.

21. Can you use a *guess and check* approach to compute the function $f(x)$ where $f'(x)=\sin(3x)$? Check your answer. How? Think about *reversing the Chain Rule here...*

22. CHALLENGE: Can you use a *guess and check* approach to compute the function $f(x)$ where $f'(x)=\frac{\sec x\tan x}{\sqrt{\sec x+8}}$ and $f(0)=7$? Check your answer.

Turn in your own solutions into Gradescope before 11:59 pm today, Tuesday Feb 13

Finish at least through number 18