Worksheet 9 Answer Key

1. $f(x) = \dfrac{x}{e^{3x}}$

$$f'(x) = \frac{e^{3x}(1) - x \cdot 3e^{3x}}{(e^{3x})^2} = \frac{\cancel{e^{3x}}(1-3x)}{(e^{3x})^{\cancel{2}}} = \frac{1-3x}{e^{3x}} \overset{\text{set}}{=} 0 \Rightarrow 1 - 3x = 0$$

$\hookrightarrow x = \frac{1}{3}$ Critical point

Sign Testing into $f'(x)$

	0		$\frac{1}{3}$		1
f'		$+$		$-$	
f		$\nearrow$	Local Max	$\searrow$	

Local Max @ $x = \frac{1}{3}$ with Max Value

$$f(1/3) = \frac{1/3}{e^{3 \cdot 1/3}} = \frac{\frac{1}{3}}{\frac{e}{1}} = \frac{1}{3e}$$

Local Min: None

2. $f(x) = \sqrt{\cos(x^2 + e^x)} + \cos\sqrt{x^2 + e^x} + e^{\sqrt{x^2 + \cos x}}$

$$f'(x) = \frac{1}{2\sqrt{\cos(x^2+e^x)}} \cdot \left(-\sin(x^2+e^x)\right) \cdot (2x + e^x) - \sin\sqrt{x^2+e^x} \cdot \frac{1}{2\sqrt{x^2+e^x}} \cdot (2x + e^x) \ldots$$

Continued $\ldots + e^{\sqrt{x^2+\cos x}} \cdot \dfrac{1}{2\sqrt{x^2+\cos x}} \cdot (2x - \sin x)$

3. $f(x) = \dfrac{1 + e^{-2x}}{1 - e^{7x}}$ Quotient Rule

$$f'(x) = \frac{(1-e^{7x}) \cdot e^{-2x} \cdot (-2) \cancel{-} (1+e^{-2x})(\cancel{-}e^{7x}) \cdot 7}{(1-e^{7x})^2}$$

FOIL

$$= \frac{-2e^{-2x} + 2e^{5x} + 7e^{7x} + 7e^{5x}}{(1-e^{7x})^2}$$

$$= \boxed{\frac{7e^{7x} + 9e^{5x} - 2e^{-2x}}{(1-e^{7x})^2}}$$

4. $$\int e^x (1 + e^x)^2 \, dx = \int u^2 \, du = \frac{u^3}{3} + C = \boxed{\frac{(1+e^x)^3}{3} + C}$$

$$\boxed{\begin{aligned} u &= 1 + e^x \\ du &= e^x \, dx \end{aligned}}$$

5. $$\int \frac{(1+e^x)^2}{e^x} \, dx = \int \frac{1 + 2e^x + e^{2x}}{e^x} \, dx = \int \frac{1}{e^x} + \frac{2\cancel{e^x}}{\cancel{e^x}} + \frac{e^{2x}}{e^x} \, dx$$

split

$$= \int e^{-x} + 2 + e^x \, dx$$

K-rule

$$= \frac{e^{-x}}{-1} + 2x + e^x + C$$

$$= \boxed{-\frac{1}{e^x} + 2x + e^x + C}$$

Try u-sub ~~$u = 1 + e^x$, $du = e^x dx$~~ NO MATCH!

$\hookrightarrow$ FOIL + Algebra

Recall

K-rule: $\displaystyle\int e^{kx} \, dx = \frac{e^{kx}}{k} + C$ (constant)

6. $\int (e^x+e^{-x})(e^x-e^{-x})\,dx = \int e^{2x} \cancel{-e^0} + \cancel{e^0} - e^{-2x}\,dx$

cancel: -1, $+1$

Method 1: Algebra FOIL

$= \frac{e^{2x}}{2} \cancel{-} \frac{e^{-2x}}{\cancel{-}2} + C$

$= \boxed{\frac{e^{2x}}{2} + \frac{e^{-2x}}{2} + C}$

Equivalent if FOIL and absorb cross term constant into $+C$ piece.

OR//

Method 2: u-sub

$$\boxed{\begin{aligned} u &= e^x+e^{-x} \\ du &= e^x-e^{-x}\,dx \end{aligned}}$$

$= \int u\,du = \frac{u^2}{2} + C = \boxed{\frac{(e^x+e^{-x})^2}{2} + C}$

$= \frac{e^{2x} + 2\cancel{e^0} + e^{-2x}}{2} + C$ (1)

$= \frac{e^{2x}+e^{-2x}}{2} + C$ Match

7. $\int (e^{4x}+e^{-9x})^2\,dx = \int (e^{4x}+e^{-9x})(e^{4x}+e^{-9x})\,dx$

FOIL Algebra

$= \int e^{8x} + e^{-5x} + e^{-5x} + e^{-18x}\,dx$

$= \int e^{8x} + 2e^{-5x} + e^{-18x}\,dx$

$= \boxed{\frac{e^{8x}}{8} + \frac{2e^{-5x}}{-5} + \frac{e^{-18x}}{-18} + C}$ OR// $= \boxed{\frac{e^{8x}}{8} - \frac{2}{5e^{5x}} - \frac{1}{18e^{18x}} + C}$

8. $\int \frac{\sqrt{1+e^{-3x}}}{e^{3x}}\,dx = -\frac{1}{3}\int \sqrt{u}\,du = -\frac{1}{3}\frac{u^{3/2}}{3/2} + C = -\frac{2}{9}(1+e^{-3x})^{3/2} + C$

($\sqrt{u} = u^{1/2}$; $\frac{1}{3/2} = \frac{2}{3}$)

$$\boxed{\begin{aligned} u &= 1+e^{-3x} \\ du &= -3e^{-3x}\,dx \\ -\frac{1}{3}\,du &= \frac{1}{e^{3x}}\,dx \end{aligned}}$$

9. $\int \frac{e^{1/x}}{x^2}\,dx = -\int e^u\,du = -e^u + C = -e^{\frac{1}{x}} + C$

$$\boxed{\begin{aligned} u &= \frac{1}{x} \\ du &= \frac{-1}{x^2}\,dx \\ -du &= \frac{1}{x^2}\,dx \end{aligned}}$$

10. $\displaystyle\int \cos x \cdot e^{5+\sin x}\,dx = \int e^{u}\,du = e^{u}+C = \boxed{e^{5+\sin x} + C}$

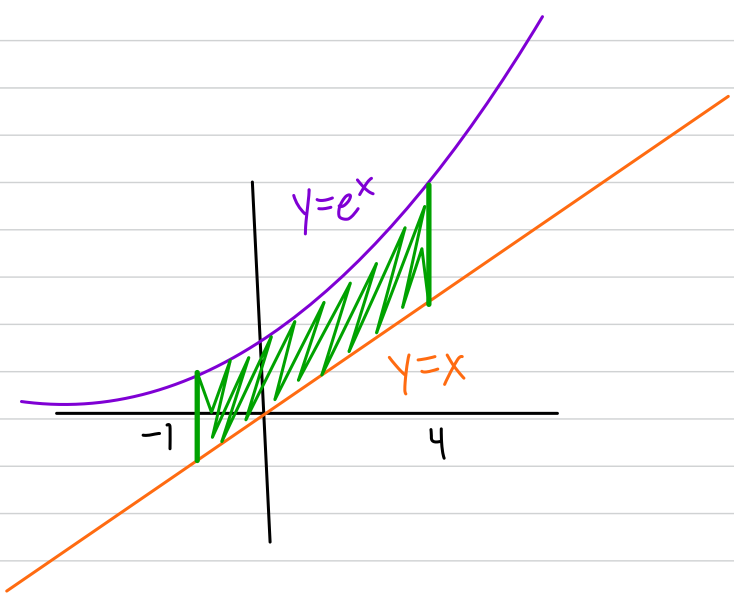
$$\boxed{\begin{array}{l} u = 5+\sin x \\ du = \cos x\,dx \end{array}}$$

e^{-x} $\quad e^{-8x}$ prep

11. $\displaystyle\int e^{8x}+e^{x}+\frac{1}{e^{x}}+\frac{1}{e^{8x}}\,dx = \frac{e^{8x}}{8}+e^{x}+\frac{e^{-x}}{-1}+\frac{e^{-8x}}{-8}+C$

$$= \boxed{\frac{e^{8x}}{8}+e^{x}-\frac{1}{e^{x}}-\frac{1}{8e^{8x}}+C}$$

12.



$\displaystyle \text{Area} = \int_{-1}^{4} \text{TOP} - \text{BOTTOM}\,dx$

$\displaystyle = \int_{-1}^{4} e^{x}-x\,dx$

$\displaystyle = e^{x}-\frac{x^{2}}{2}\Big|_{-1}^{4}$

$\displaystyle = e^{4}-\overset{8}{\cancel{\frac{16}{2}}}-\left(e^{-1}-\frac{1}{2}\right)$

$\displaystyle = e^{4}-\underset{-\frac{16}{2}}{8}-\frac{1}{e}+\frac{1}{2} = \boxed{e^{4}-\frac{1}{e}-\frac{15}{2}}$

13. $\displaystyle f'(x) = \frac{e^{3x}}{\sqrt{8+e^{3x}}}$ $\qquad f(0) = -4$

$\displaystyle f(x) = \int f'(x)\,dx = \int \frac{e^{3x}}{\sqrt{8+e^{3x}}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{3}\int u^{-1/2}\,du \rightarrow \frac{u^{1/2}}{1/2}$

$$\boxed{\begin{array}{l} u = 8+e^{3x} \\ du = 3e^{3x}dx \\ \frac{1}{3}\,du = e^{3x}dx \end{array}}$$

$\displaystyle = \frac{1}{3}\cdot 2\sqrt{u}+C = \frac{2}{3}\sqrt{8+e^{3x}}+C$

Test $\displaystyle f(0) = \frac{2}{3}\sqrt{8+e^{0}}+C \overset{\text{set}}{=} -4$ $\qquad \sqrt{9}=3$, $e^0 = 1$

$\displaystyle \frac{2}{\cancel{3}}\cdot\cancel{3}+C = -4$

$2+C = -4$

$\hookrightarrow C = -6$

Finally, $\displaystyle \boxed{f(x) = \frac{2}{3}\sqrt{8+e^{3x}}-6}$

14. $f'(x) = \dfrac{x^2}{e^{x^3}}$ and $f(2) = \dfrac{1}{e^8}$

K-rule $\boxed{\int e^{kx}\,dx = \dfrac{e^{kx}}{k} + c}$

K-rule

$$f(x) = \int f'(x)\,dx = \int \frac{x^2}{e^{x^3}}\,dx = \frac{1}{3}\int \frac{1}{e^u}\,du = \frac{1}{3}\int e^{-u}\,du = \frac{1}{3}\cdot\frac{e^{-u}}{-1} + c$$

$$\boxed{\begin{aligned} u &= x^3 \\ du &= 3x^2\,dx \\ \tfrac{1}{3}\,du &= x^2\,dx \end{aligned}}$$

$$= \frac{-1}{3e^u} + c = -\frac{1}{3e^{x^3}} + C$$

Test $x = 2$

$$f(2) = -\frac{1}{3e^{(2^3)}} + C = -\frac{1}{3e^8} + c \overset{\text{set}}{=} \frac{1}{e^8}$$

$$C = \frac{3 \cdot 1}{3e^8} + \frac{1}{3e^8} = \frac{4}{3e^8}$$

Finally, $f(x) = \boxed{\dfrac{-1}{3e^{(x^3)}} + \dfrac{4}{3e^8}}$

15. $f(x) = \displaystyle\int f'(x)\,dx = \int \frac{e^{\sqrt{\tan x}} \cdot \sec^2 x}{\sqrt{\tan x}}\,dx = 2\int e^u\,du = 2e^u + C$

$$\boxed{\begin{aligned} u &= \sqrt{\tan x} \\ du &= \frac{1}{2\sqrt{\tan x}} \cdot \sec^2 x\,dx \\ 2\,du &= \frac{1}{\sqrt{\tan x}} \cdot \sec^2 x\,dx \end{aligned}}$$

$$= 2e^{\sqrt{\tan x}} + C$$

1-2e

Test: $f\left(\frac{\pi}{4}\right) = 2e^{\sqrt{\tan\frac{\pi}{4}}} + C \overset{\text{set}}{=} 1$

1

1

$2e + C = 1$

$\Rightarrow C = 1 - 2e$

Finally, $f(x) = \boxed{2e^{\sqrt{\tan x}} + (1 - 2e)}$