

Worksheet 7 Answer Key

$$\begin{aligned}
 (a) \int_1^5 7 - x - x^2 dx &= 7x - \frac{x^2}{2} - \frac{x^3}{3} \Big|_1^5 = 35 - \frac{25}{2} - \frac{125}{3} - \left(7 - \frac{1}{2} - \frac{1}{3}\right) \\
 &= 35 - \frac{25}{2} - \frac{125}{3} - 7 + \frac{1}{2} + \frac{1}{3} = 28 - \frac{24}{2} - \frac{124}{3} = 16 - \frac{124}{3} = \frac{48}{3} - \frac{124}{3} = -\frac{76}{3}
 \end{aligned}$$

$$(b) \int_1^5 7 - x - x^2 dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(1 + \frac{4i}{n}\right) \cdot \frac{4}{n}$$

Here $f(x) = 7 - x - x^2$

$a = 1$ $b = 5$

$\Delta x = \frac{b-a}{n} = \frac{5-1}{n} = \frac{4}{n}$

$x_i = a + i\Delta x$

$= 1 + i\left(\frac{4}{n}\right)$

$= 1 + \frac{4i}{n}$

$$= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n f\left(1 + \frac{4i}{n}\right)$$

$$= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n 7 - \left(1 + \frac{4i}{n}\right) - \left(1 + \frac{4i}{n}\right)^2$$

$$= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n 7 - 1 - \frac{4i}{n} - 1 - \frac{8i}{n} - \frac{16i^2}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n 5 - \frac{12i}{n} - \frac{16i^2}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n 5 - \sum_{i=1}^n \frac{12i}{n} - \sum_{i=1}^n \frac{16i^2}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n 5 - \frac{4}{n} \sum_{i=1}^n 12i - \frac{4}{n} \sum_{i=1}^n \frac{16i^2}{n}$$

$$= \lim_{n \rightarrow \infty} \frac{20}{n} \sum_{i=1}^n 1 - \frac{48}{n^2} \sum_{i=1}^n i - \frac{64}{n^3} \sum_{i=1}^n i^2$$

$$= \lim_{n \rightarrow \infty} \frac{20}{n} \cdot n - \frac{48}{n^2} \left(\frac{n(n+1)}{2}\right) - \frac{64}{n^3} \frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n \rightarrow \infty} 20 - \frac{48}{2} \left(\frac{n}{n}\right) \left(\frac{n+1}{n}\right) - \frac{64}{6} \left(\frac{n}{n}\right) \left(\frac{n+1}{n}\right) \left(\frac{2n+1}{n}\right)$$

$$= \lim_{n \rightarrow \infty} 20 - 24(1) \left(1 + \frac{1}{n}\right) - \frac{64}{6} (1) \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right)$$

$$= 20 - 24(1)(1) - \frac{64}{6} (1)(1)(2)$$

$$= 20 - 24 - \frac{64}{3}$$

$$= -4 - \frac{64}{3} = -\frac{12}{3} - \frac{64}{3} = -\frac{76}{3}$$

$$2. \quad g(x) = \int_x^2 \frac{\cos t}{5 + \cos t} dt = - \int_2^x \frac{\cos t}{5 + \cos t} dt$$

$$g'(x) = \frac{d}{dx} \left(- \int_2^x \frac{\cos t}{5 + \cos t} dt \right) = - \frac{\cos x}{5 + \cos x}$$

$$3. \quad \int_0^4 \frac{1-x}{\sqrt{x}} dx = \int_0^4 \left(\frac{1}{\sqrt{x}} - \frac{x}{\sqrt{x}} \right) dx = \int_0^4 x^{-1/2} - x^{1/2} dx = \left. \frac{x^{1/2}}{1/2} - \frac{x^{3/2}}{3/2} \right|_0^4$$

$$= 2\sqrt{x} - \frac{2}{3}x^{3/2} \Big|_0^4 = 2\sqrt{4} - \frac{2}{3} \cdot 4^{3/2} - (0 - 0)$$

$$= 4 - \frac{16}{3} = \frac{12}{3} - \frac{16}{3} = -\frac{4}{3}$$

$$4. \quad \int x^4 (2 - 3x^5)^6 dx = -\frac{1}{15} \int u^6 du = -\frac{1}{15} \cdot \frac{u^7}{7} + C = -\frac{(2 - 3x^5)^7}{105} + C$$

$$\begin{aligned} u &= 2 - 3x^5 \\ du &= -15x^4 dx \\ -\frac{1}{15} du &= x^4 dx \end{aligned}$$

$$5. \quad \int_9^{64} \frac{5}{\sqrt{x}\sqrt{1+\sqrt{x}}} dx = 5 \cdot 2 \int_4^9 \frac{1}{\sqrt{u}} du = \left. \frac{10 u^{1/2}}{1/2} \right|_4^9 = 20\sqrt{u} \Big|_4^9$$

$$= 20(\sqrt{9} - \sqrt{4})$$

$$\begin{aligned} u &= 1 + \sqrt{x} \\ du &= \frac{1}{2\sqrt{x}} dx \\ 2 du &= \frac{1}{\sqrt{x}} dx \end{aligned}$$

$$\begin{aligned} x=9 &\Rightarrow u = 1 + \sqrt{9} = 4 \\ x=64 &\Rightarrow u = 1 + \sqrt{64} = 9 \end{aligned}$$

$$= 20 \cdot 1 = 20$$

$$6. \int_0^{\frac{\pi}{6}} \frac{\cos x}{(1+6\sin x)^2} dx = \frac{1}{6} \int_1^4 \frac{1}{u^2} du = \frac{1}{6} \int_1^4 u^{-2} du = \frac{1}{6} \left(\frac{u^{-1}}{-1} \right) \Big|_1^4 = -\frac{1}{6u} \Big|_1^4$$

$$\begin{aligned} u &= 1 + 6\sin x \\ du &= 6\cos x dx \\ \frac{1}{6} du &= \cos x dx \end{aligned}$$

$$\begin{aligned} x=0 &\Rightarrow u=1+6\sin 0=1 \\ x=\frac{\pi}{6} &\Rightarrow u=1+6\sin \frac{\pi}{6} \\ &= 1+3=4 \end{aligned}$$

$$= -\frac{1}{24} + \frac{1}{6} = -\frac{1}{24} + \frac{4}{24} = \frac{3}{24} = \frac{1}{8}$$

$$7. \int \frac{5}{x^2 \left(5 + \frac{3}{x}\right)^{\frac{3}{5}}} dx = 5 \cdot \left(-\frac{1}{3}\right) \int \frac{1}{u^{\frac{2}{5}}} du = -\frac{5}{3} \int u^{-\frac{3}{5}} du$$

$$= -\frac{5}{3} \frac{u^{\frac{2}{5}}}{\frac{2}{5}} + C$$

$$= -\frac{25}{6} \left(5 + \frac{3}{x}\right)^{\frac{2}{5}} + C$$

$$\begin{aligned} u &= 5 + \frac{3}{x} \rightarrow 3x^{-1} \\ du &= -3x^{-2} dx \\ -\frac{1}{3} du &= \frac{1}{x^2} dx \end{aligned}$$

Challenge:

$$8. \int x(x-2)^{\frac{3}{4}} dx = \int (u+2)u^{\frac{3}{4}} du = \int u^{\frac{7}{4}} + 2u^{\frac{3}{4}} du$$

$$\begin{aligned} u &= x-2 \Rightarrow x=u+2 \\ du &= dx \end{aligned}$$

$$= \frac{u^{\frac{11}{4}}}{\frac{11}{4}} + \frac{2u^{\frac{7}{4}}}{\frac{7}{4}} + C$$

$$= \frac{4}{11} (x-2)^{\frac{11}{4}} + \frac{8}{7} (x-2)^{\frac{7}{4}} + C$$

$$9. f(x) = \int \frac{\sec^2 x}{\sqrt{3+\tan x}} dx = \int \frac{1}{\sqrt{u}} du = \int u^{-\frac{1}{2}} du = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$\begin{aligned} u &= 3 + \tan x \\ du &= \sec^2 x dx \end{aligned}$$

$$= 2\sqrt{u} + C = 2\sqrt{3+\tan x} + C$$

$$\text{Test } f\left(\frac{\pi}{4}\right) = 2\sqrt{3+\tan \frac{\pi}{4}} + C = -5$$

$$2\sqrt{4} + C = -5 \Rightarrow C = -9$$

Finally, $f(x) = 2\sqrt{3+\tan x} - 9$

$$10 \quad v(t) = \sin t$$

$$(a) \quad a(t) = v'(t) = \cos t$$

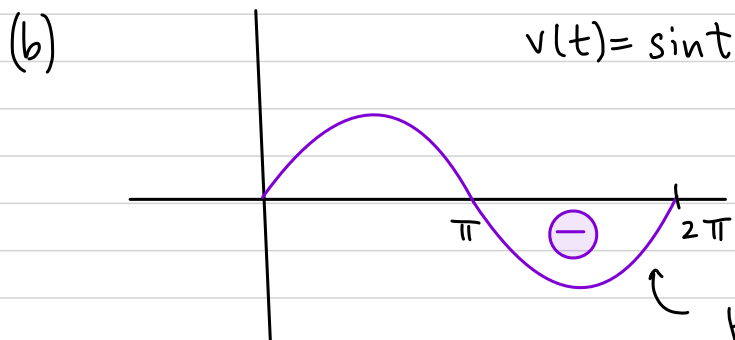
and

$$s(t) = \int v(t) dt = \int \sin t dt = -\cos t + C$$

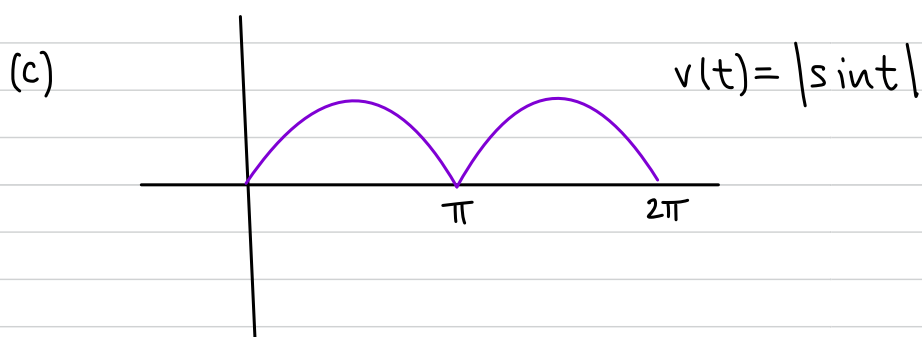
$$\text{Test } S(0) = -\cancel{\cos} 0 + C = 2 \quad \Rightarrow C = 3$$

set
-1 + C = 2

$$\hookrightarrow s(t) = -\cos t + 3$$



Here $v(t) = \sin t$ negative which means
Position is decreasing, which means
the object is moving to the left
for time $t = \pi$ to $t = 2\pi$



$$(d) \quad \text{Displacement} = \int_0^{2\pi} v(t) dt = \int_0^{2\pi} \sin t dt \quad \dots \text{FTC}$$

$$\text{Total Distance} = \int_0^{2\pi} |v(t)| dt = \int_0^{2\pi} |\sin t| dt \quad \dots \text{Split cases + FTC on each piece}$$