

Worksheet 6 Answer Key

$$1. \int x^7 \cdot (4-x^8)^6 dx = -\frac{1}{8} \int u^6 du = -\frac{1}{8} \cdot \frac{u^7}{7} + C = -\frac{u^7}{56} + C$$

$$\begin{aligned} u &= 4-x^8 \\ du &= -8x^7 dx \\ -\frac{1}{8} du &= x^7 dx \end{aligned}$$

$$= -\frac{(4-x^8)^7}{56} + C$$

$$2. \int \frac{\cos \sqrt{x}}{\sqrt{x}} dx = 2 \int \cos u du = 2 \sin u + C = 2 \sin \sqrt{x} + C$$

$$\begin{aligned} u &= \sqrt{x} \\ du &= \frac{1}{2\sqrt{x}} dx \\ 2 du &= \frac{1}{\sqrt{x}} dx \end{aligned}$$

$$3. \int \cos x \cdot \sin^6 x dx = \int \cos x (\sin x)^6 dx = \int u^6 du = \frac{u^7}{7} + C = \frac{\sin^7 x}{7} + C$$

$$\begin{aligned} u &= \sin x \\ du &= \cos x dx \end{aligned}$$

$$4. \int x^5 \sqrt{x^6+7} dx = \frac{1}{6} \int \sqrt{u} du = \frac{1}{6} \int u^{\frac{1}{2}} du = \frac{1}{6} \cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$\begin{aligned} u &= x^6+7 \\ du &= 6x^5 dx \\ \frac{1}{6} du &= x^5 dx \end{aligned}$$

$$= \frac{1}{9} u^{\frac{3}{2}} + C = \frac{1}{9} (x^6+7)^{\frac{3}{2}} + C$$

$$5. \int \frac{x}{(x^2+1)^9} dx = \frac{1}{2} \int \frac{1}{u^9} du = \frac{1}{2} \int u^{-9} du = \frac{1}{2} \frac{u^{-8}}{-8} + C$$

$$\begin{aligned} u &= x^2+1 \\ du &= 2x dx \\ \frac{1}{2} du &= x dx \end{aligned}$$

$$= -\frac{1}{16u^8} + C = -\frac{1}{16(x^2+1)^8} + C$$

$$6. \int \frac{\sin x}{\cos^5 x} dx = \int \frac{\sin x}{(\cos x)^5} dx = - \int \frac{1}{u^5} du = - \int u^{-5} du = \cancel{\frac{u^{-4}}{-4}} + C$$

$$\begin{aligned} u &= \cos x \\ du &= -\sin x dx \\ -du &= \sin x dx \end{aligned}$$

$$= \frac{1}{4 u^4} + C = \frac{1}{4 \cos^4 x} + C$$

$$7. \int \sec^2 x \cdot \tan^5 x dx = \int \sec^2 x \cdot (\tan x)^5 dx = \int u^5 du = \frac{u^6}{6} + C = \frac{\tan^6 x}{6} + C$$

$$\begin{aligned} u &= \tan x \\ du &= \sec^2 x dx \end{aligned}$$

$$8. \int \frac{(9 + \frac{1}{x})^3}{x^2} dx = - \int u^3 du = -\frac{u^4}{4} + C = -\frac{(9 + \frac{1}{x})^4}{4} + C$$

$$\begin{aligned} u &= 9 + \frac{1}{x} \\ du &= -\frac{1}{x^2} dx \\ -du &= \frac{1}{x^2} dx \end{aligned}$$

$$9. \int_1^4 \frac{1}{\sqrt{x} (1+\sqrt{x})^3} dx = 2 \int_2^3 \frac{1}{u^3} du = 2 \int_2^3 u^{-3} du = \cancel{2} \frac{u^{-2}}{-2} \Big|_2^3 = -\frac{1}{u^2} \Big|_2^3$$

$$\begin{aligned} u &= 1 + \sqrt{x} \\ du &= \frac{1}{2\sqrt{x}} dx \\ 2du &= \frac{1}{\sqrt{x}} dx \end{aligned}$$

$$\begin{aligned} x=1 &\Rightarrow u = 1 + \sqrt{1} = 2 \\ x=4 &\Rightarrow u = 1 + \sqrt{4} = 3 \end{aligned}$$

$$= -\frac{1}{3^2} - \left(-\frac{1}{2^2}\right)$$

$$= -\frac{1}{9} + \frac{1}{4} = \frac{-4}{36} + \frac{9}{36} = \frac{5}{36}$$

$$10. f(x) = \int f'(x) dx = \int x^2 \sin(x^3) dx = \frac{1}{3} \int \sin u du = \frac{1}{3} (-\cos u) + C$$

$$\begin{aligned} u &= x^3 \\ du &= 3x^2 dx \\ \frac{1}{3} du &= x^2 dx \end{aligned}$$

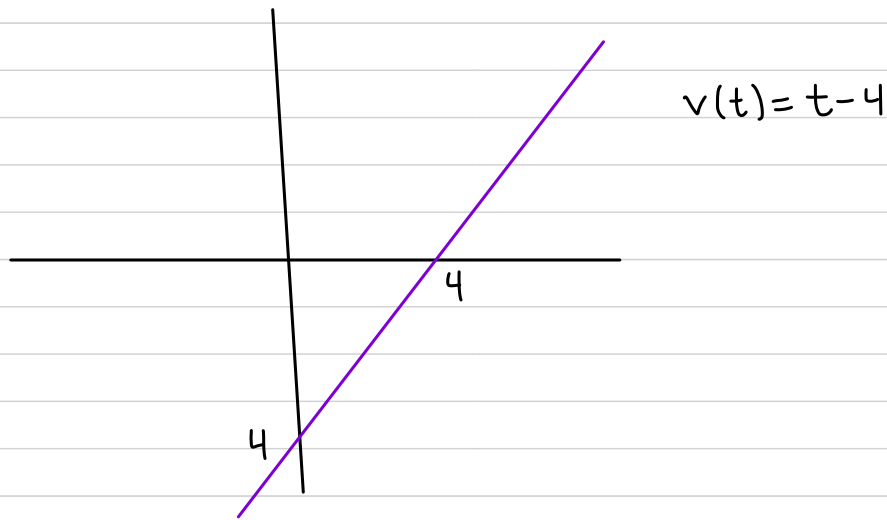
$$= -\frac{1}{3} \cos(x^3) + C$$

Test

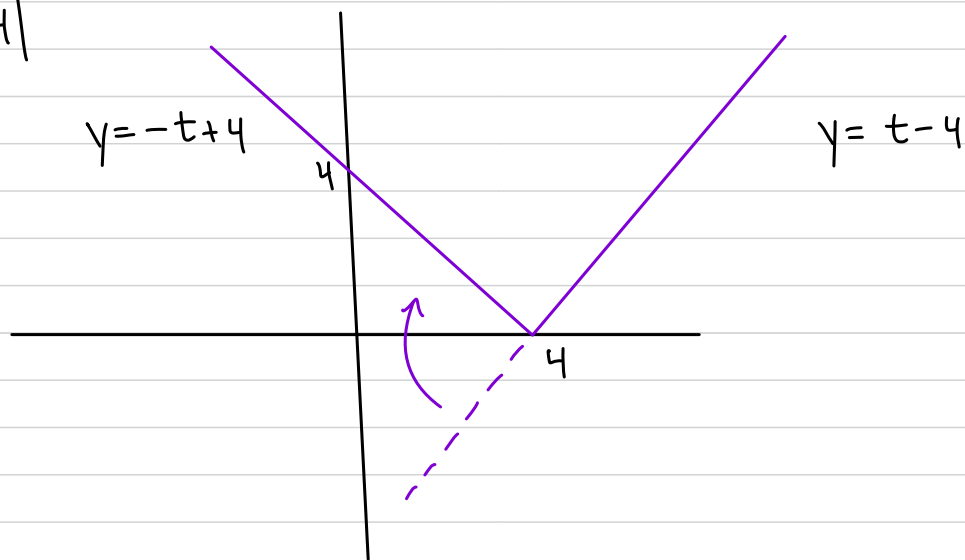
$$f(0) = -\frac{1}{3} \cos 0 + C = -\frac{1}{3} + C \stackrel{\text{set}}{=} 3 \Rightarrow C = 3 + \frac{1}{3} = \frac{10}{3}$$

$$\text{Finally, } f(x) = -\frac{1}{3} \cos(x^3) + \frac{10}{3}$$

$$11(a) v(t) = t - 4$$



$$11(b) |v(t)| = |t - 4|$$



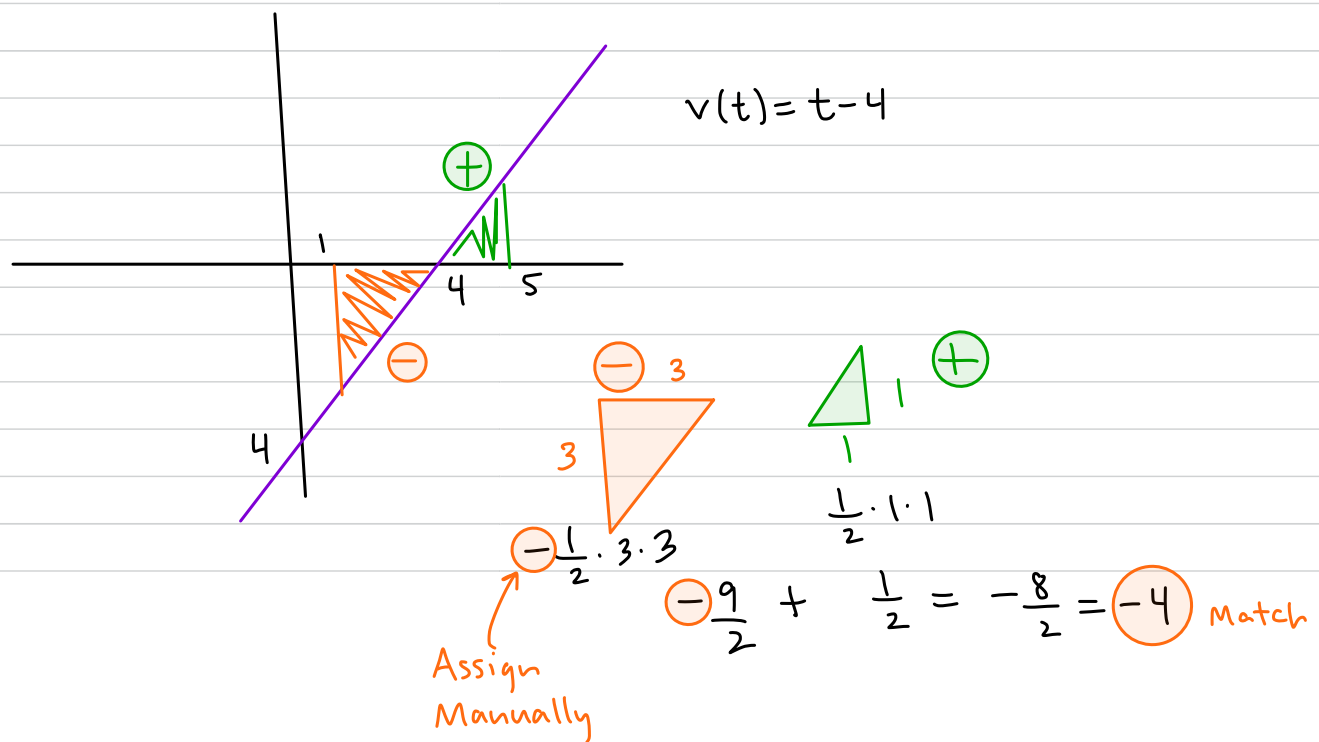
$$11(c) |v(t)| = \begin{cases} t - 4 & \text{if } t \geq 4 \\ -(t - 4) & \text{if } t < 4 \end{cases} = \begin{cases} t - 4 & \text{if } t \geq 4 \\ -t + 4 & \text{if } t < 4 \end{cases}$$

$$11(d) \text{ Displacement} = \int_1^5 v(t) dt = \int_1^5 t - 4 dt = \left. \frac{t^2}{2} - 4t \right|_1^5$$

$$= \frac{25}{2} - 20 - \left(\frac{1}{2} - 4 \right) = \frac{25}{2} - 20 - \frac{1}{2} + 4$$

$$= \frac{24}{2} - 16 = 12 - 16 = -4$$

Area Interpretations: Definite Integral = Area Above x-axis - Area Below x-axis



$$\begin{aligned}
 \text{11(e) Total Distance} &= \int_1^5 |v(t)| dt = \int_1^4 |v(t)| dt + \int_4^5 |v(t)| dt \\
 &= \int_1^4 -t+4 dt + \int_4^5 t-4 dt \\
 &= -\frac{t^2}{2} + 4t \Big|_1^4 + \frac{t^2}{2} - 4t \Big|_4^5
 \end{aligned}$$

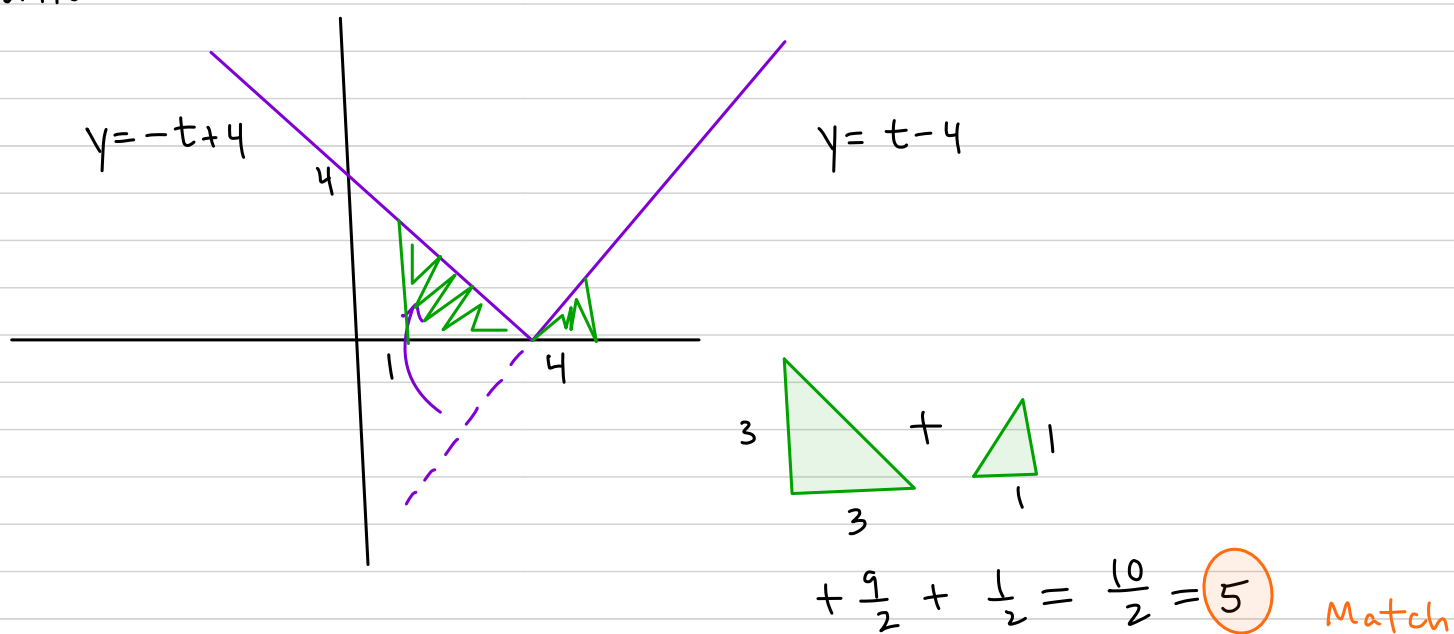
$$= \frac{-16}{2} + 16 - \left(-\frac{1}{2} + 4\right) + \frac{25}{2} - 20 - \left(\frac{16}{2} - 16\right)$$

$$= -8 + 16 + \frac{1}{2} - 4 + \frac{25}{2} - 20 - 8 + 16$$

$\frac{26}{2} = 13$

$$= 16 + 13 - 24 = 29 - 24 = \textcircled{5} \quad \text{should be } \textcircled{+}$$

Area Interpretation



$$12. \quad f'(x) = \frac{d}{dx} \int_5^x \frac{1}{t+7} dt = \frac{1}{x+7}$$

$$13. \quad f'(x) = \frac{d}{dx} \int_x^9 \sqrt{t^2+3} dt = \frac{d}{dx} \left(- \int_9^x \sqrt{t^2+3} dt \right) = -\sqrt{x^2+3}$$

$$14. \quad \text{First } g'(x) = \frac{d}{dx} \int_x^7 \sqrt{1+\cos t} dt = \frac{d}{dx} \left(- \int_7^x \sqrt{1+\cos t} dt \right) = -\sqrt{1+\cos x}$$

$$g''(x) = \cancel{\frac{1}{2\sqrt{1+\cos x}}} (\cancel{-\sin x}) = \frac{\sin x}{2\sqrt{1+\cos x}}$$