

Worksheet 3 Answer Key

$$1(a) \quad f(x) = \sin \left(\cos^6 \left(\frac{9}{x^8} \right) \right) \stackrel{\text{prep}}{=} \sin \left[\cos \left(\frac{9}{x^8} \right) \right]^6 \quad \begin{matrix} 9x^{-8} \\ \textcircled{2} \end{matrix}$$

$$f'(x) = \cos \left(\cos^6 \left(\frac{9}{x^8} \right) \right) \cdot 6 \left(\cos \left(\frac{9}{x^8} \right) \right)^5 \cdot \left[-\sin \left(\frac{9}{x^8} \right) \right] \cdot \left(-72x^{-9} \right)$$

$$1(b) \quad y = \tan \left(\frac{9}{\sin x} \right) \stackrel{\text{prep}}{=} \tan \left[9 (\sin x)^{-1} \right]$$

$$y' = \sec^2 \left(\frac{9}{\sin x} \right) \cdot 9 \left[-(\sin x)^{-2} \right] \cdot \cos x$$

OR

Chain Rule + Quotient Rule

$$y' = \sec^2 \left(\frac{9}{\sin x} \right) \cdot \left(\frac{\sin x \cdot 0 - 9 \cdot \cos x}{\sin^2 x} \right) = \sec^2 \left(\frac{9}{\sin x} \right) \cdot \left(\frac{-9 \cos x}{\sin^2 x} \right)$$

Match. O.K.

$$1(c) \quad g(t) = \frac{8 + \sec(7t^2)}{9 + \cos t}$$

$$g'(t) = \frac{(9 + \cos t) (\sec(7t^2) \cdot \tan(7t^2) \cdot (14t) - (8 + \sec(7t^2)) \cdot (-\sin t))}{(9 + \cos t)^2}$$

Do Not Simplify

$$2. \quad f(x) = \cos^2(2x) + \tan(2x) + \sin(6x) + \sqrt{3} \cdot x \quad \text{Find } f'\left(\frac{\pi}{6}\right)$$

prep $\rightarrow (\cos(2x))^2$

$$f'(x) = 2 \cos(2x) (-\sin(2x)) \cdot 2 + \sec^2(2x) \cdot 2 + 6 \cos(6x) + \sqrt{3}$$

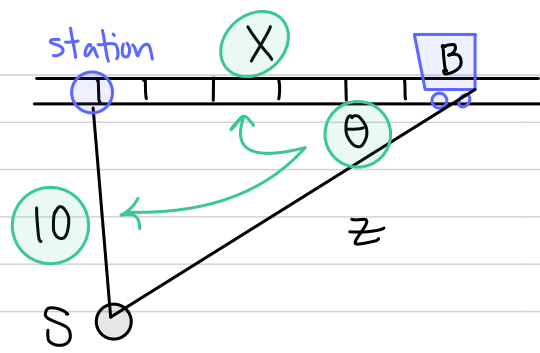
$$= -4 \cos(2x) \sin(2x) + 2 \sec^2(2x) + 6 \cos(6x) + \sqrt{3}$$

$$f'\left(\frac{\pi}{6}\right) = -4 \cos\left(2 \cdot \frac{\pi}{6}\right) \sin\left(2 \cdot \frac{\pi}{6}\right) + 2 \left(\sec\left(2 \cdot \frac{\pi}{6}\right) \right)^2 + 6 \cos\left(6 \cdot \frac{\pi}{6}\right) + \sqrt{3}$$

$$= -4 \cos\left(\frac{\pi}{3}\right) \sin\left(\frac{\pi}{3}\right) + 2 \left(\sec\left(\frac{\pi}{3}\right) \right)^2 + 6 \cos(\pi) + \sqrt{3}$$

$$= -4 \left(\frac{1}{2} \right) \left(\frac{\sqrt{3}}{2} \right) + 2 \cdot 4 - 6 + \sqrt{3} = -\sqrt{3} + 8 - 6 + \sqrt{3} = 2$$

3. Diagram



Variables

Let x = Distance between Train and Station

z = Distance between Bob (on Train) and Sally

θ = Angle of Bob's Head Rotation from Track Line

Given $\frac{d\theta}{dt} = -2$ Radians/second

Find $\frac{dx}{dt} = ?$ when $z = 13$ meters

Equation

$$\tan \theta = \frac{10}{x}$$

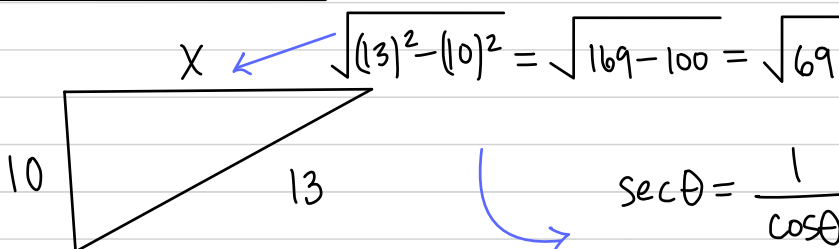
Differentiate

$$\frac{d}{dt} (\tan \theta) = \frac{d}{dt} (10x^{-1})$$

$$\sec^2 \theta \cdot \frac{d\theta}{dt} = -10x^{-2} \frac{dx}{dt}$$

$$(\sec \theta)^2 \frac{d\theta}{dt} = \frac{-10}{x^2} \cdot \frac{dx}{dt}$$

Extra Solvable Information



$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\left(\frac{A}{H}\right)} = \frac{H}{A} = \frac{13}{\sqrt{69}}$$

Substitute

$$\left(\frac{13}{\sqrt{69}}\right)^2 \cdot (-2) = -\frac{10}{(\sqrt{69})^2} \cdot \frac{dx}{dt}$$

$$\frac{169}{69} (-2) = \frac{-10}{69} \cdot \frac{dx}{dt}$$

Solve

$$\frac{dx}{dt} = \frac{169}{69} (-2) \cdot \frac{69}{-10} = \frac{169}{5} \text{ m/sec}$$

Answer

The train is travelling $\frac{169}{5}$ meters every second at that moment.

7. $f''(x) = 20x^3 + 12x^2 + 4$ $f(0) = 8$ and $f(1) = 5$

$$f'(x) = \int f''(x) dx = \int 20x^3 + 12x^2 + 4 dx$$

$$= \frac{20x^4}{4} + \frac{12x^3}{3} + 4x + C$$

constant

note: no f' info given

$$f(x) = \int f'(x) dx = \int 5x^4 + 4x^3 + 4x + C$$

$$= \frac{5x^5}{5} + \frac{4x^4}{4} + \frac{4x^2}{2} + Cx + D$$

New Constant!

★ Use both initial conditions to solve for C and D

$$= x^5 + x^4 + 2x^2 + Cx + D$$

$$f(0) = \underbrace{0+0+0+0}_{\text{all 0}} + D = 8 \Rightarrow D = 8$$

use simplest one 1st

$$f(1) = \underbrace{1+1+2}_{4} + C + 8 = 5 \quad C + 12 = 5 \Rightarrow C = -7$$

Finally, $f(x) = x^5 + x^4 + 2x^2 - 7x + 8$

8(a) $\int \frac{7x^{2/5} + 8x^{-4/2} + \frac{1}{x}}{\sqrt{x}} dx = \int \frac{7x^{2/5}}{x^{1/2}} + \frac{8x^{-4/2}}{x^{1/2}} + \frac{x^{-1}}{x^{1/2}} dx$

split-split

$$= \int \frac{7x^{4/10}}{x^{5/10}} + \frac{8x^{-8/6}}{x^{3/6}} + \frac{x^{-2/2}}{x^{1/2}} dx$$

$$= \int 7x^{\frac{4}{10}-\frac{5}{10}} + 8x^{\frac{-8}{6}-\frac{3}{6}} + x^{\frac{-2}{2}-\frac{1}{2}} dx$$

$$= \frac{10}{9} \left(\frac{7x^{9/10}}{9/10} + \frac{8x^{-5/6}}{-5/6} + \frac{x^{-3/2}}{-1/2} \right) + C$$

note: All Algebra Prep

Recall: $\frac{x^a}{x^b} = x^{a-b}$

$$= \frac{70}{9} x^{9/10} - \frac{48}{5} x^{-5/6} - 2x^{-1/2} + C$$

or $= \frac{70}{9} x^{9/10} - \frac{48}{5x^{5/6}} - \frac{2}{\sqrt{x}} + C$

$$8(b) \int \left(\sqrt{3} + \frac{1}{x^3} \right) \left(x - \frac{1}{x^{2/7}} \right) dx = \int \sqrt{3}x - \frac{\sqrt{3}}{x^{2/7}} + \frac{x}{x^3} - \frac{1}{x^3 \cdot x^{2/7}} dx$$

$$x^3 \cdot x^{2/7} = x^{21/7} x^{2/7} = x^{23/7}$$

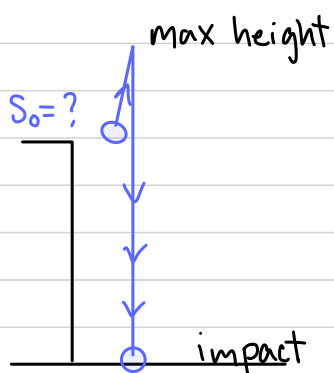
$$= \int \sqrt{3}x - \sqrt{3}x^{-2/7} + x^{-2} - x^{-23/7} dx \quad \text{prep}$$

$$= \frac{\sqrt{3}x^2}{2} - \frac{\sqrt{3}x^{5/7}}{5/7} + \frac{x^{-1}}{-1} - \frac{x^{-16/7}}{-16/7} + C$$

flip

$$= \frac{\sqrt{3}x^2}{2} - \frac{7\sqrt{3}}{5}x^{5/7} - \frac{1}{x} + \frac{7}{16}x^{-16/7} + C$$

9.



Equations of Motion

$$a(t) = -32$$

$$v(t) = -32t + v_0$$

$$s(t) = -16t^2 + v_0 t + S_0$$

Given: $v(0) = 80 \text{ ft/sec}$

$v(t_{\text{impact}}) = -112 \text{ ft/sec}$

Find: $S_0 = ?$

Impact:

$$v(t_{\text{impact}}) = -32t + 80 = -112$$

$$32t = 192$$

$$t_{\text{impact}} = \frac{192}{32} = 6 \text{ seconds}$$

Plug $t_{\text{impact}} = 6$ seconds into position and set equal to 0

That is, $s(t_{\text{impact}}) = s(6) = 0$

$$s(6) = -16(6)^2 + 80(6) + S_0 = 0$$

note: S_0 only unknown

$$-576 + 480 + S_0 = 0$$

$$-96 + S_0 = 0$$

$$\Rightarrow S_0 = 96 \text{ feet}$$

$$\begin{array}{r} 3 \\ 36 \\ 16 \\ \hline 216 \\ 360 \\ \hline 576 \end{array}$$

Answer: The building is 96 feet tall