1. $\boxed{\displaystyle\int x^n\, dx = \frac{x^{n+1}}{n+1} + C}$ $(n \neq -1)$

2. $\displaystyle\int \sqrt{x}\, dx = \int x^{1/2}\, dx = \frac{x^{3/2}}{3/2} + C = \boxed{\frac{2}{3} x^{3/2} + C}$

3. $\displaystyle\int \frac{1}{x^9}\, dx \overset{\text{prep}}{=} \int x^{-9}\, dx = \frac{x^{-8}}{-8} + C = \boxed{-\frac{1}{x^8} + C}$

4. $\displaystyle\int \frac{1}{\sqrt{x}}\, dx \overset{\text{prep}}{=} \int x^{-1/2}\, dx = \frac{x^{1/2}}{1/2} + C = \boxed{2\sqrt{x} + C}$

5. $\displaystyle\int \frac{1}{x^{3/7}}\, dx \overset{\text{prep}}{=} \int x^{-\frac{3}{7}}\, dx = \frac{x^{4/7}}{\frac{4}{7}} + C = \boxed{\frac{7}{4} x^{4/7} + C}$

6. $\displaystyle\int \cos x\, dx = \boxed{\sin x + C}$

7. $\displaystyle\int \sin x\, dx = \boxed{-\cos x + C}$

8. $\displaystyle\int \sec^2 x\, dx = \boxed{\tan x + C}$

9. $\displaystyle\int \sec x \cdot \tan x\, dx = \boxed{\sec x + C}$

10. $f'(x) = \dfrac{d}{dx}\left(\dfrac{3}{4} x + x^{3/4} - \dfrac{1}{x^{3/4}} + \underset{\text{Constant}}{\boxed{\dfrac{4}{3}}} + \dfrac{1}{x^{4/3}} + \dfrac{3}{4x^4} - \dfrac{4}{x^3} \right)$

$\overset{\text{prep}}{=} \dfrac{d}{dx}\left(\dfrac{3}{4} x + x^{3/4} - x^{-3/4} + \dfrac{4}{3} + x^{-4/3} + \dfrac{3}{4} x^{-4} - 4x^{-3} \right)$

$= \dfrac{3}{4} + \dfrac{3}{4} x^{-1/4} + \dfrac{3}{4} x^{-7/4} + 0 - \dfrac{4}{3} x^{-7/3} + \dfrac{3}{4}(-4 x^{-5}) + 12 x^{-4}$

$= \boxed{\dfrac{3}{4} + \dfrac{3}{4} x^{-1/4} + \dfrac{3}{4} x^{-7/4} - \dfrac{4}{3} x^{-7/3} - 3x^{-5} + 12 x^{-4}}$

OR $= \boxed{\dfrac{3}{4} + \dfrac{3}{4x^{1/4}} + \dfrac{3}{4x^{7/4}} - \dfrac{4}{3x^{7/3}} - \dfrac{3}{x^5} + \dfrac{12}{x^4}}$

11. $\int f(x)\,dx = \int \frac{3}{4}x + x^{3/4} - \frac{1}{x^{3/4}} + \frac{4}{3} + \frac{1}{x^{4/3}} + \frac{3}{4x^4} - \frac{4}{x^3}\,dx$

$\overset{\text{prep}}{=}$ (as above in #10) $\int \frac{3}{4}x + x^{3/4 \, +1} - x^{-3/4 \, +1} + \frac{4}{3} + x^{-4/3 \, +1} + \frac{3}{4}x^{-4 \, +1} - 4x^{-3 \, +1}\,dx$

$= \frac{3}{4}\cdot\frac{x^2}{2} + \frac{x^{7/4}}{7/4} - \frac{x^{1/4}}{1/4} + \frac{4}{3}\cdot x + \frac{x^{-1/3}}{-1/3} + \frac{3}{4}\cdot\frac{x^{-3}}{-3} - 4\cdot\frac{x^{-2}}{-2} + C$

Simplify

$= \boxed{\frac{3x^2}{8} + \frac{4}{7}x^{7/4} - 4x^{1/4} + \frac{4}{3}x - 3x^{-1/3} - \frac{x^{-3}}{4} + 2x^{-2} + C}$

OR

$= \left(\frac{3x^2}{8} + \frac{4}{7}x^{7/4} - 4x^{1/4} + \frac{4}{3}x - \frac{3}{x^{1/3}} - \frac{1}{4x^3} + \frac{2}{x^2} + C \right)$

12. $\int x^3(1+x^2)\,dx = \int x^3 + x^5\,dx = \left(\frac{x^4}{4} + \frac{x^6}{6} + C \right)$

13. $\int \frac{x + \sqrt{x} + 7}{x^3}\,dx = \int \frac{x}{x^3} + \frac{\sqrt{x}}{x^3} + \frac{7}{x^3}\,dx$ (Split-Split) $x^{1/2}$ $x^{3} \, x^{1/2}$

OR $\int (x + x^{1/2} + 7)\,x^{-3}\,dx$
⋮

$\overset{\text{prep}}{=} \int x^{-2} + x^{-5/2} + 7x^{-3}\,dx$

$= \frac{x^{-1}}{-1} + \frac{x^{-3/2}}{-3/2} + \frac{7x^{-2}}{-2} + C$

$= \left(-x^{-1} - \frac{2}{3}x^{-3/2} - \frac{7}{2}x^{-2} + C \right)$

OR $= \left(-\frac{1}{x} - \frac{2}{3x^{3/2}} - \frac{7}{2x^2} + C \right)$

14. $\int x^2 + x(1+x)^2\, dx = \int x^2 + x(1+x)(1+x)\, dx$ **FOIL Algebra**

$= \int x^2 + x(1 + 2x + x^2)\, dx$ **Distribute**

$= \int x^2 + x + 2x^2 + x^3\, dx$

Combine + Simplify

$= \int x^3 + 3x^2 + x\, dx$

$= \dfrac{x^4}{4} + \dfrac{3x^3}{3} + \dfrac{x^2}{2} + C$

$= \dfrac{x^4}{4} + x^3 + \dfrac{x^2}{2} + C$

15. $\int -3\cos x - \sec^2 x - 7\sec x \tan x - \sin x\, dx$

$= -3(\sin x) - \tan x - 7(\sec x) - (-\cos x) + C$

$= -3\sin x - \tan x - 7\sec x + \cos x + C$

16. $y = 2x + \sin x$ Horizontal Tangent Line means Derivative equals 0
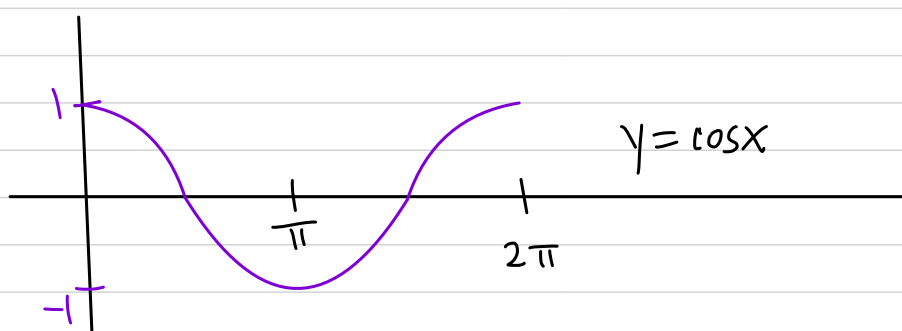
$y' = 2 + \cos x \overset{\text{set}}{=} 0$

Solve $\cos x = -2$ which has no solution

not possible   ↳ Recall Range of cosine is $[-1, 1]$

OR think about Unit Circle.

y = cos x

17. $f(x) = \int f'(x)\, dx = \int 2 + \sin x \; dx$

$$= 2x - \cos x + C \qquad \leftarrow \text{the Most General Antiderivative}$$

Test Initial Value

$$f\left(\tfrac{\pi}{2}\right) = 2\!\!\!\!\diagup\left(\tfrac{\pi}{2}\right) - \cos\!\!\!\!\diagup\left(\tfrac{\pi}{2}\right)^{\!\!0} + C \overset{\text{Set}}{=} 3$$

$$\pi + C = 3 \quad \overset{\text{Solve}}{\Rightarrow} \quad C = 3 - \pi \qquad \text{Plug back in...}$$

Finally, $f(x) = \boxed{2x - \cos x + 3 - \pi}$

18. $f(x) = \int f'(x)\, dx = \int x^2 + 1 \; dx = \dfrac{x^3}{3} + x + C$

Test Initial Value

$$f(1) = \tfrac{1}{3} + 1 + C \overset{\text{set}}{=} 3$$

$$\tfrac{4}{3} + C = 3 \quad \overset{\text{Solve}}{\Rightarrow} \quad C = 3 - \tfrac{4}{3} = \tfrac{9}{3} - \tfrac{4}{3} = \tfrac{5}{3} \qquad \text{Plug back in...}$$

Finally, $f(x) = \boxed{\dfrac{x^3}{3} + x + \dfrac{5}{3}}$

19. $f(x) = \int f'(x)\, dx = \int x\left(2 + \sqrt{x}\right) dx = \int 2x + x^{3/2}\, dx$

$$= \frac{\diagup\!\!\!2\, x^2}{\diagup\!\!\!2} + \frac{x^{5/2}}{5/2} + C$$

$$= \boxed{x^2 + \dfrac{2}{5} x^{5/2} + C}$$

Test Initial Value

$$f(4) = 16 + \tfrac{2}{5}(4)^{5/2} + C \overset{\text{set}}{=} 30$$

$$16 + \tfrac{2}{5}\left(\sqrt{4}\right)^{5 \nearrow 32} + C = 30$$

$$16 + \dfrac{64}{5} + C = 30 \quad \overset{\text{Solve}}{\Rightarrow} \quad C = \underbrace{30 - 16}_{} - \dfrac{64}{5}$$

$$= 14 - \dfrac{64}{5}$$

$$= \dfrac{70}{5} - \dfrac{64}{5} = \dfrac{6}{5} \qquad \text{Plug back in}$$

Finally, $f(x) = \boxed{x^2 + \dfrac{2}{5} x^{5/2} + \dfrac{6}{5}}$

20. $f'(x) = \int f''(x)\,dx = \int \frac{1}{\sqrt{x}} + 3x^2\,dx$

$\text{prep} \to x^{-1/2}$

$$= \frac{x^{1/2}}{\frac{1}{2}} + \frac{3x^3}{3} + C$$

$$= 2\sqrt{x} + x^3 + C_1 \qquad \text{use one } f' \text{ condition to find } C_1$$

Test Initial Value

$$f'(1) = 2\cdot\sqrt{1} + 1 + C_1 \overset{\text{set}}{=} 2$$

$$2 + 1 + C_1 = 2 \quad \overset{\text{Solve}}{\Rightarrow} \quad C_1 = -1$$

Collect so far:

$$f'(x) = 2\sqrt{x} + x^3 - 1$$

Next:

$$f(x) = \int f'(x)\,dx = \int 2x^{1/2} + x^3 - 1\,dx$$

$$\overset{=\frac{2}{3}}{=} 2\cdot\frac{x^{3/2}}{\frac{3}{2}} + \frac{x^4}{4} - x + C_2 \qquad \text{use other } f \text{ value to find } C_2$$

$$= \frac{4}{3}x^{3/2} + \frac{x^4}{4} - x + C_2$$

Test Value

$$f(1) = \frac{4}{3}(1)^{3/2} + \frac{1}{4} - 1 + C_2 \overset{\text{set}}{=} 0$$

$$\frac{4}{3} + \frac{1}{4} - 1 + C_2 = 0$$

$$\frac{16}{12} + \frac{3}{12} - \frac{12}{12} + C_2 = 0 \Rightarrow C_2 = \frac{-7}{12}$$

$$\frac{7}{12}$$

Finally, $f(x) = \frac{4}{3}x^{3/2} + \frac{x^4}{4} - x - \frac{7}{12}$

GUESS AND CHECK

21. $f(x) = \dfrac{-\cos(3x)}{3}$ because $\dfrac{d}{dx}\left(-\dfrac{1}{3}\cos(3x)\right) = -\dfrac{1}{3}\left(-\sin(3x)\cdot 3\right) = \sin(3x)$ ✓

22. $f(x) = 2\sqrt{\sec x + 8} + C = 2(\sec x + 8)^{1/2} + C$

because $f'(x) = \cancel{2} \cdot \left(\dfrac{1}{\cancel{2}\sqrt{\sec x + 8}}\right) \cdot (\sec x \cdot \tan x) = \dfrac{\sec x \tan x}{\sqrt{\sec x + 8}}$ Match ✓

Now Test Value

$f(0) = \underbrace{2}\sqrt{\underbrace{(\sec 0)}_{1} + 8} + C \overset{\text{Set}}{=} 7$

$\underbrace{\sqrt{9} \to 3}$

$\sec 0 = \dfrac{1}{\cos 0} = 1$

$6 + C = 7 \implies C = 1$

Finally, $f(x) = \boxed{2\sqrt{\sec x + 8} + 1}$