Math 106, Benedetto

What you need to know for Exam 3

This Exam covers Chapter 6 on the Natural Exponential and Logarithm. We studied their graphs, domains and ranges, algebra, values, as well as their relevant Limits, Derivatives and Integrals. Along the way, we took care to organize their Chain Rules and Antiderivatives, including the k-rule. For Integration, we paid particular attention to the three typical strategies: elementary antiderivative, algebra or u-substitution. We also covered Area between curves in Section 5.1. The following is a list of most of the topics covered. **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.** Remember, no calculators in any exams.

- 6.2: Exponentials. Know the definition of a^x for any positive $a > 0$. Know the basic properties of exponential functions. Most importantly, know the definition of the number e, the derivative of e^x, Chain Rule, the algebra and properties of e^x, and the antiderivative of e^x and e^{kx} where k is a constant. Key: $e^0 = 1$.

- 6.1 Inverses. Know what 1-1 functions are. Know the definition of the Inverse function. Know how to find the graph of Inverse functions.

- 6.3: Logarithms. Know the definition, algebra and basic properties of the Natural Logarithms $\ln x$. Key: $\ln 1 = 0$ but $\ln 0$ is **undefined**. Know the inverse properties $e^{\ln x} = x$ and $\ln e^x = x$. Practice simplifying $e^{-2\ln 3}$ or $\ln e^{-7}$.

- 6.4: Derivatives of Logarithms. Know the derivative of $\ln x$ and the antiderivative $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$. Remember the Absolute Value and $+C$. Know the Chain Rule too. Know how to use the Algebraic Properties of the Log to compute complex Log Derivatives. Also, Logarithmic Differentiation. You should be able to compute the derivative of, say, x^x or $x^{\sin x}$. Note: we do not know the antiderivative of $\ln x$.

- 5.1: Areas between curves. Know how to sketch a region when told its bounding curves. We only integrated with respect to x. Be able to determine limits of integration (i.e., where the region starts and ends), also determine where the functions intersect, as well as knowing which function is on top and which is on bottom.

  We will still be relying on u-substitution.

- 4.5: Integration using u-Substitution. Know the substitution rule for indefinite integrals and for definite integrals. Practice **a lot** to make sure you can use them. Make sure to **change** (or mark) your limits of integration in u-substitution.

Techniques of Integration

1. "We Know It" Snap facts, MAKE A NEW LONGER LIST
2. Algebra
3. u-substitution

Common Types of Problems

- Know the graphs of e^x and $\ln x$. Also some key values. Be efficient here.
- Know how to compute derivatives, including Exponential rule, and Chain Rule. Know when it is a Power rule, an Exponential rule or a Constant Derivative. Know when the function has a variable or a constant formula. Note: e^x is a function and e^5 is a constant value. Make sure to use same Algebra Prep as before.
- Know how to compute a Definite Integral involving Exponentials and Logs using FTC Part II. Know how to simplify combinations of exponential and log values. Review the k-rule, it is for Antiderivatives.
- Solve Initial Valued Differential Equations for a specific Antiderivative. For this Exam, the antidifferentiation may involve u-substitution. Use the given value to solve for $+C$.
- Compute derivatives of *variable limit* integrals using the Fundamental Theorem of Calculus Part I.
- Compute integrals involving u-substitution for **both** Indefinite (which gains $+C$) and Definite Integrals (which requires a **change of the limits** using the choice of u).
- Compute derivatives of a Logarithm holding a large and complex input chunk. Use Log Algebra to simplify before Differentiating.
- Compute derivatives using Logarithmic Differentiation. You use this when the function has a variable in the base *and* the power, like x^x or $(\sin x)^x$. Or simpler, for other exponentials like $f(x) = 3^x$. Pratice proving $\dfrac{d}{dx}a^x = a^x \ln a$ where $a > 0$ is constant.
- Solve equations involving Exponentials or Logarithms, like $e^{6-5x} = 7$ or $\ln(4x-7) = 9$.
- Compute Absolute Maximum or Minimum values for Exponential and Log funtions.
- Compute Tangent Line Equations or values where Horizontal Tangents exist.
- Compute Area bounded between curves. Make sure to sketch the graphs and shade the bounded regions. Make sure to solve for what x-values the curves intersect, if they are not given in the problem.
- As before, practice computing integrals, both definite and indefinite. Remember, you have two main tools: you can simplify the integrand using algebra, and/or you can do a substitution. The idea is to reduce the integrand to something you actually know the antiderivative of. The key thing is to **keep trying different ideas until something works**. Practice a combined technique of speed and accuracy!!!

  TIP: Remember if you are given a function involving a Product or Quotient of powers of x then you must simplify using algebra and break it down into simpler pieces and finish using Power Rules. Always **prep** your powers to the numerator using exponential algebra. Review all exponential algebra for simplifying your powers. When it's time to evaluate using the limits of integration, it's advised to move the negative power back into the denominator using exponential algebra rules, *before* evaluating at the two endpoints.