Math 106, Spring 2024

Homework #9

Due **Friday, March 1st** in Gradescope by 11:59 pm ET

Goal: More Warm-Up Algebra for future Area Computations

Recall the Series Summation Rules

$$\sum_{i=1}^{n} 1 = \underbrace{1+1+1+\ldots+1}_{\text{n copies}} = n \qquad \sum_{i=1}^{n} \text{constant} = \text{constant} \overbrace{\sum_{i=1}^{n} 1}^{n} = \text{constant} \cdot n$$

$$\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i \qquad \sum_{i=1}^{n} \text{constant} \cdot a_i = \text{constant} \sum_{i=1}^{n} a_i$$

For problems 1-6, Simplify each Series sum in terms of i and/or n. Recall that you can pull n and constants out of the Series Sums but not i, because i is the counter index for the Series.

1. Show that $\displaystyle\sum_{i=1}^{n} 7 = \boxed{7n}$

2. Show that $\displaystyle\sum_{i=1}^{n} (-5) = \boxed{-5n}$

3. Show that $\displaystyle\sum_{i=1}^{n} \left(\frac{5i}{n} - 7\right) \cdot \left(\frac{3}{n}\right) = \boxed{\left(\frac{15}{n^2} \sum_{i=1}^{n} i\right) - 21}$

4. Show that $\displaystyle\sum_{i=1}^{n} \left(2 + \frac{5i}{n}\right)^2 \cdot \left(\frac{4}{n}\right) = \boxed{\left(\frac{100}{n^3} \sum_{i=1}^{n} i^2\right) + \left(\frac{80}{n^2} \sum_{i=1}^{n} i\right) + 16}$

5. Show that

$$\sum_{i=1}^{n} \left[\left(-1 + \frac{4i}{n}\right)^2 - 3\left(-1 + \frac{4i}{n}\right) - 8\right] \cdot \left(\frac{4}{n}\right) = \boxed{\left(\frac{64}{n^3} \sum_{i=1}^{n} i^2\right) - \left(\frac{80}{n^2} \sum_{i=1}^{n} i\right) - 16}$$

6. Show that

$$\sum_{i=1}^{n} \left[\left(-2 + \frac{3i}{n}\right)^2 - 5\left(-2 + \frac{3i}{n}\right) + 6\right] \cdot \left(\frac{3}{n}\right) = \boxed{\left(\frac{27}{n^3} \sum_{i=1}^{n} i^2\right) - \left(\frac{81}{n^2} \sum_{i=1}^{n} i\right) + 60}$$

REGULAR OFFICE HOURS

Monday: 12:00–3:00 pm

Tuesday: 1:00–4:00 pm

7:30–9:00 pm TA Alexa, SMUDD 208A

Wednesday: 1:00-3:00 pm

Thursday: none for Professor

6:00–7:30 pm TA Alexa, SMUDD 208A

Friday: 12:00–2:00 pm

- Email with any questions `dbenedetto@amherst.edu`
- Please present a Final Draft only.