Math 106, Spring 2024

**Homework #3**

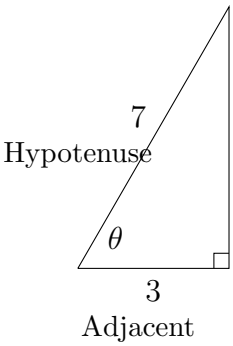
Due **Wednesday, February 7th** in Gradescope by 11:59 pm ET

**Goal:** More Trigonometry, Angles & Trigonometric Derivatives (including the Chain Rule).

**FIRST:** Read through and understand the following Examples.

Ex: Consider an angle $\theta$ where $0 \leq \theta < \frac{\pi}{2}$. Suppose that $\cos\theta = \frac{3}{7}$. Find the value for both $\sin\theta$ and $\tan\theta$.

Here, $\cos\theta = \frac{3}{7} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ and we can set up the related triangle using the given Trig ratio




? $\sqrt{7^2 - 3^2} = \sqrt{49 - 9} = \sqrt{40}$
Opposite

As a result, we can read the sine and tangent off this reference triangle.

$$\sin\theta = \boxed{\frac{\sqrt{40}}{7}} = \frac{\text{Opposite}}{\text{Hypotenuse}} \quad \text{and} \quad \tan\theta = \boxed{\frac{\sqrt{40}}{3}} = \frac{\text{Opposite}}{\text{Adjacent}}$$

Ex: Consider $F(x) = \tan(2x) - \sin(3x)$. Compute $F'\left(\frac{\pi}{6}\right)$.

First, compute the derivative

$F'(x) = \sec^2(2x) \cdot 2 - \cos(3x) \cdot 3 = 2\sec^2(2x) - 3\cos(3x)$

Next, evaluate the derivative at the specific value $\frac{\pi}{6}$.

$$F'\left(\frac{\pi}{6}\right) = 2\sec^2\left(2\left(\frac{\pi}{6}\right)\right) - 3\cos\left(3\left(\frac{\pi}{6}\right)\right) = 2\sec^2\left(\frac{\pi}{3}\right) - 3\cos\left(\frac{\pi}{2}\right)$$

$$= \frac{2}{\cos^2\left(\frac{\pi}{3}\right)} - 3\cancel{\cos\left(\frac{\pi}{2}\right)}^{0} = \frac{2}{\left(\cancel{\cos}\left(\frac{1}{2}\pi\right)\right)^2} - 0 = \frac{2}{\left(\frac{1}{2}\right)^2} = \frac{2}{\frac{1}{4}} = 2 \cdot \frac{4}{1} = \boxed{8}$$

**Next, Complete the following Homework problems.**

For $\#1-2$, evaluate the following Trig expressions, keeping $0 \le \theta < \dfrac{\pi}{2}$

1. If $\sin\theta = \dfrac{1}{2}$, find $\cos\theta$ 2. If $\cos\theta = \dfrac{2}{5}$, find $\tan\theta$

For $\#3-4$, use the facts $\boxed{\dfrac{d}{dx}\sin x = \cos x}$ and $\boxed{\dfrac{d}{dx}\cos x = -\sin x}$ to prove that

3. $\boxed{\dfrac{d}{dx}\tan x = \sec^2 x}$ and 4. $\boxed{\dfrac{d}{dx}\sec x = \sec x \tan x}$ **Memorize.**

For $\#5-6$, solve for angle(s) $\theta$ in Radians keeping $0 \le \theta < 2\pi$.

5. $\sin\theta = -\dfrac{1}{2}$ 6. $\sin\theta = -\dfrac{\sqrt{3}}{2}$

For $\#7-8$, compute the following values. Justify. Show work on the Unit Circle/Trig Triangles.

7. $\cos\dfrac{4\pi}{3}$ 8. $\sin\dfrac{4\pi}{3}$

For $\#9-17$, compute the Derivative for each of the following functions. Do **Not** simplify.

9. $y = \sin(x^2 - 5x + 8)$ 10. $f(x) = \sin^2 x$ 11. $y = \cos^6(3x)$

12. $y = \cos\sqrt{x}$ 13. $y = \sqrt{\cos x}$ 14. $f(x) = \dfrac{\cos(3x)}{\sin(4x)}$

15. $y = \tan\left(\dfrac{1}{x}\right)$ 16. $f(x) = \dfrac{1}{\tan x}$ 17. $y = \left(\dfrac{\cos x}{x^2 - \sin x}\right)^8$

18. Let $G(x) = \sin(2x) - \cos(3x)$. Compute $G'\left(\dfrac{\pi}{6}\right)$. Simplify your answer completely.

# REGULAR OFFICE HOURS

## Monday: 12:00–3:00 pm

## Tuesday: 1:00–4:00 pm

**7:30–9:00 pm TA Alexa, SMUDD 208a**

## Wednesday: 1:00-3:00 pm

## Thursday: none for Professor

**6:00–7:30 pm TA Alexa, SMUDD 208a**

## Friday: 12:00–2:00 pm

• We've finished a solid review of Trigonometry, and derivatives from Math 105. Aim to make clearer and neater solutions this week.

• Attend Office Hours regularly, both with Professor Benedetto and Math Fellow Alexa Martinez.